## Liouville's Theorem :

→ "The density of distribution $\rho$ is a function of position co-ordinates and momentum co-ordinates."

$$\boxed{\rho = \rho(q, p, t)}$$

→ If the state of an ensemble change with time, the partition of phase points in the phase space change with time.

→ The motion of these phase points in the phase space is governed by the canonical equations.

$$\boxed{\dot{q}_i = \frac{\partial H}{\partial p_i}, \quad \dot{p}_i = \frac{-\partial H}{\partial q_i}}$$

Here, $H = H(q, p, t)$ is the Hamiltonian of the system.

→ Due to motion of phase points, the density of states in the phase space changes with time.

→ Liouville's Theorem gives information about the rate of change of phase density in the phase space.

This theorem consists of two parts,

i) Principle of conservation of density in the phase space :

→ "The rate of change of density of phase points in the neighbourhood of a moving phase point in $\Gamma$ space is zero."

→ Mathematically this may be represented as,

$$\frac{d\rho}{dt} = 0$$

## ii) Principle of conservation of extension in phase space :

→ "Any arbitrary elements of volume (or) extension in phase in the $\Gamma$ space, bounded by a moving surface and containing a number of phase points doesn't change with time despite the displacements and distortions"

→ Mathematically this may be represented as

$$\boxed{\frac{d}{dt}(\delta\tau) = 0}$$

→ $$\left(\frac{\partial \rho}{\partial t}\right)_{q,p} + \sum_{i=1}^{f} \frac{\partial \rho}{\partial q_i}\dot{q}_i + \sum_{i=1}^{f} \frac{\partial \rho}{\partial p_i}\cdot\dot{p}_i = 0$$

$$\boxed{\left[\frac{\partial \rho}{\partial t}\right]_{q,p} = -\sum_{i=1}^{f}\left[\frac{\partial \rho}{\partial q_i}\dot{q}_i + \frac{\partial \rho}{\partial p_i}\dot{p}_i\right]}$$

→ It explain the phase density $\rho$

## Statistical Equilibrium :

→ An ensemble is said to be in statistical equilibrium, if it obey the conditions,

$$\boxed{\left[\frac{\partial \rho}{\partial t}\right]_{q,p} = 0}$$

→ This holds good for all values of q & P.

→ Density ($\rho$) is to be independent of time at all points in the phase space for an ensemble in statistical equilibrium.

## Law of Equipartition of Energy:

→ It states that, the total K.E of a dynamical system consisting of a large no. of particles in thermal equilibrium is equally divided among its all the degrees of freedom and average K.E associated with each degree of freedom is $\frac{1}{2}KT$.

Where, K – Boltzmann Constant

T – Absolute temp. of the system.

→ Average energy $\propto \frac{1}{2}KT$

Average energy $\propto$ Absolute Temperature.