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Laplace Transformation

TRB - Mathematics

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Mathematics Material for

- TNSET
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Theory of equations

Laplace transformation

Laplace transformation directly gives the solution of differential equations with given initial conditions without finding the general solution and then evaluating the arbitrary constants.

Def: Let $f(t)$ be a function of t defined for all $t \geq 0$. Then the laplace transform of $f(t)$ is denoted by $L(f(t))$, and it is defined by

$$L(f(t)) = \int_0^\infty e^{-st} f(t)\,dt$$

where s is real or complex parameter.

Linear property:

$$L\{c_1 f(t) + c_2 g(t)\} = c_1 L(f(t)) + c_2 L(g(t))$$

Laplace transform of some elementary transformations:

1. $L(1) = \frac{1}{s}$
2. $L(t^n) = \frac{n!}{s^{n+1}}$
3. $L(e^{at}) = \frac{1}{s-a}$
4) $L(e^{-at}) = \frac{1}{s+a}$
5) $L(\sin at) = \frac{a}{s^2+a^2}$
6) $L(\cos at) = \frac{s}{s^2+a^2}$
7) $L(\sinh at) = \frac{a}{s^2-a^2}$
8) $L(\cosh at) = \frac{s}{s^2-a^2}$

First shifting Theorem:

If $L\{f(t)\} = \bar{f}(s)$ then $L(e^{at}f(t)) = \bar{f}(s-a)$

Results

1. $L(e^{at}t^n) = \dfrac{n!}{(s-a)^{n+1}}$

2. $L(e^{at}\sin bt) = \dfrac{b}{(s-a)^2+b^2}$

3. $L(e^{at}\cos bt) = \dfrac{s-a}{(s-a)^2+b^2}$

4. $L(e^{at}\sinh bt) = \dfrac{b}{(s-a)^2+b^2}$

5) $L(e^{at}\cosh bt) = \dfrac{s-a}{(s-a)^2 - b^2}$

Note: $L(e^{-at}f(t)) = \bar{f}(s+a)$

Results:

1. $L(e^{-at}t^n) = \dfrac{n!}{(s+a)^{n+1}}$

2. $L(e^{-at}\sin bt) = \dfrac{b}{(s+a)^2+b^2}$

3. $L(e^{-at}\cos bt) = \dfrac{s+a}{(s+a)^2+b^2}$

4. $L(e^{-at}\sinh bt) = \dfrac{b}{(s+a)^2+b^2}$

5) $L(e^{-at}\cosh bt) = \dfrac{s+a}{(s+a)^2 - b^2}$

Change of scale property:

If $L(f(t)) = \bar{f}(s)$ then

$L[f(at)] = \dfrac{1}{a}\bar{f}(s/a)$

Laplace transform of Derivatives:

If $L[f(t)] = \bar{f}(s)$ then

1) $L[f'(t)] = s\,L(f(t)) - f(0)$

2) $L[f''(t)] = s^2 \bar{f}(s) - s f(0) - f'(0)$

3) $L(f^n(t)) = s^n \bar{f}(s) - s^{n-1} f(0) - s^{n-2} f'(0) - \dots - s f^{n-2}(0) - f^{n-1}(0)$

Laplace transform of Integrals:

1. If $L[f(t)] = \bar{f}(s)$ then

$$L\left[\int_0^t f(u)\,du\right] = \frac{\bar{f}(s)}{s}$$

2. If $L[f(t)] = \bar{f}(s)$ then

$$L[t\,f(t)] = -\frac{d}{ds}\bar{f}(s)$$

3. $L[t^2 f(t)] = (-1)^2 \frac{d^2}{ds^2}\bar{f}(s)$

4) $L[t^n f(t)] = (-1)^n \frac{d^n}{ds^n}\bar{f}(s)$

Inverse Linear transformation:

1. $L^{-1}(1/s) = 1$

2. $L^{-1}\left(\frac{1}{s-a}\right) = e^{at}$

3. $L^{-1}\left(\frac{1}{s^n}\right) = \frac{t^{n-1}}{(n-1)!}$ if n is +ve.

4. $L^{-1}\left\{\frac{1}{(s-a)^n}\right\} = e^{at}\frac{t^{n-1}}{(n-1)!}$

5. $L^{-1}\left\{\frac{1}{s^2+a^2}\right\} = \frac{1}{a}\sin at$

6. $L^{-1}\left\{\frac{s}{s^2+a^2}\right\} = \cos at$

7. $L^{-1}\left\{\frac{1}{s^2-a^2}\right\} = \frac{1}{a}\sinh at$

8. $L^{-1}\left\{\frac{s}{s^2-a^2}\right\} = \cosh at$

9) $L^{-1}\left\{\frac{1}{(s-a)^2+b^2}\right\} = \frac{1}{b}e^{at}\sin bt$

10) $L^{-1}\left\{\frac{s-a}{(s-a)^2+b^2}\right\} = e^{at}\cos bt$.

Division by t:

$$L\left\{\frac{1}{t}f(t)\right\} = \int_s^\infty e^{-st} f(t)\, ds.$$

Convolution theorem:

If $L^{-1}\{\bar{f}(s)\} = f(t)$ and

$L^{-1}\{\bar{g}(s)\} = g(t)$

Then $L^{-1}\{\bar{f}(s)\,\bar{g}(s)\} = \int_0^t f(u)\, g(t-u)\, du$.

Application to Differential equations:

Solving differential equations with using laplace transformation:

1. Take laplace transformations of both sides of the given differential equations, using initial conditions.
2. This gives an algebraic equation.
3. Solve the algebraic equation to get L(y) interms of s.
4. Taking laplace transformation of both sides.
5. This gives y as function of t which is desired solution.