TIRUVANNAMALAI

Binomial series

UG-TRB Mathematics & T NPSC Statistical Inspector Exam

K.Sellavel M.sc, B.Ed Mathematics

TRB – Mathematics

Unit : 1 – Algebra.

Binomial Series

Some important results:

1. $(1-x)^{-p/q} = 1 + \frac{p}{1!}\left(\frac{x}{q}\right) + \frac{p(p+q)}{2!}\left(\frac{x}{q}\right)^2 + \frac{p(p+q)(p+2q)}{3!}\left(\frac{x}{q}\right)^3 + \cdots$

2. $(1+x)^{-p/q} = 1 - \frac{p}{1!}\left(\frac{x}{q}\right) + \frac{p(p+q)}{3!}\left(\frac{x}{q}\right)^2 - \frac{p(p+q)(p+2q)}{3!}\left(\frac{x}{q}\right)^3 + \cdots$

3. Binomial series is vaild only when $|x| < 1$

Problems:

1. The expansion of $(3x-4)^{-7}$ is valid if

(a) $|x| < 4/3$ (b) $|x| < 1$ (c) $|x| < 4$ (d) $\left|\frac{2x}{3}\right| < 1$.

Soln

$$(3x-4)^{-7} = (-4)^{-7}\left(\frac{3x}{-4}+1\right)$$

$$= (-4)^{-7}\left(1 - \frac{3}{4}x\right)$$

∴ Explansion is valid if

$$\left|\frac{3}{4}x\right| < 1 \Rightarrow |x| < 3/4.$$

∴ Ans: (a)

2. The expansion of $(1-2x)^{-1}$ is valid if

(a) $|x| < 2$ (b) $|x| < 1$ (c) $|x| < 1/2$ (d) $|x| < 0$

Soln Expansion is valid only if

$|2x| < 1$

Ans: c $\Rightarrow |x| < 1/2$

3. The expansion of $(1+3x)^{5/2}$ is valid if

(a) $|x| < \frac{1}{3}$ (b) $|x| < 2$ (c) $|x| \leq 1/3$ (d) $|x| < 5$

Soln Expansion is valid if

$|3x| < 1$

$\Rightarrow |x| < 1/3$

$\Rightarrow$ Ans: c

Some important results:

1. $(1-x)^{-1} = 1 + x + x^2 + x^3 + \cdots$
2. $(1+x)^{-1} = 1 - x + x^2 - x^3 + \cdots$

Problems:

1. The co.effi. of x^n in $(1-x)^{-1}$ is

(a) 0 (b) 1 (c) $1/n!$ (d) $n(n+1)$

Ans: b

2. The co.eff of x^n in $(1-3x)^{-1}$ is

(a) 1 (b) $\frac{3^n}{n!}$ (c) 3^n (d) $(-1)^n 3^n$

Ans: c

3. The co.eff. of x^7 in $(1+2x)^{-7}$ is.

(a) 64 (b) -64 (c) 128 (d) -128

Soln

co.eff. of $x^n = (-1)^7 (2)^7$

$= -128$

$\therefore$ Ans: d.

Steps for summation of a Binomial series:

1. The series should begin with 1, if not do adjustments.
2. Second term on words, the factor in the Numerator, as well as the factors in the denometer of each terms form an A.P.
3. The number of factor in the numerator should be the same as the number of factors in the denominator
4. Express the denominators in factorial form

P - Numerator of the 2nd term

6. Find x

7. Use the formula $(1-x)^{-P/q}$ (or) $(1+x)^{-P/q}$

Problems:

1. $1 + \frac{3}{4} + \frac{3.5}{4.8} + \frac{3.5.7}{4.8.12} \cdots =$ (a) $\sqrt{2/3}$ (b) $(2/3)^{3/2}$ (c) $\sqrt{3}$ (d) $2^{2/3}$

Soln

$S = (1+x)^{-P/q}$

$= (1+1/2)^{-3/2}$

$= (3/2)^{-3/2}$

$= (2/3)^{3/2}$

$P = 3$, $q = 2$, $(x/q) = 1/4$, $x = 2/4 = 1/2$

Ans: b

2. $1 + \frac{3}{4} + \frac{3.5}{4.8} + \frac{3.5.7}{4.8.12} \cdots =$ (a) $2\sqrt{2}$ b) $(2)^{1/3}$ (c) $\sqrt{2}/3$ (d) $2^{2/3}$

Soln

$S = (1-x)^{-P/q}$

$= (1-1/2)^{-3/2}$

$= (+1/2)^{-3/2}$

$= (2)^{3/2}$

$= 2\sqrt{2}$

$P = 3$

$q = 2$

$x/q = 1/4$

$x = 1/2$

Ans: a

3. $1 - \frac{1}{5} + \frac{1.4}{5.10} - \frac{1.4.7}{5.10.15} + \cdots =$ (a) $2\sqrt{2}$ (b) $(5/8)^{1/3}$ (c) $\frac{\sqrt{2}}{3}$ (d) $2^{2/3}$

Soln

$S = (1+x)^{-P/q}$

$= (1+3/5)^{-1/3}$

$= (8/5)^{-1/3}$

$= (5/8)^{1/3}$

$P = 1$

$q = 3$

$x/q = 1/5$

$x = 3/5$

Ans: b

4) $1 + \frac{1}{6} + \frac{1.4}{6.12} + \frac{1.4.7}{6.12.18} \cdots$ = a) $2\sqrt{2}$ b) $(2)^{1/3}$ c) $\frac{\sqrt{2}}{3}$ d) $2^{2/3}$

Soln

$S = (1-x)^{-p/q}$

$= (1-1/2)^{-1/3}$

$= (1/2)^{-1/3}$

$= (2)^{1/3}$

$p = 1, \quad q = 3$

$n/q = 1/6 \Rightarrow x = 1/2$

Ans : b

8. $1 - \frac{1}{6} + \frac{1.3}{6.12} - \frac{1.3.5}{6.12.18} + \cdots$ = a) $2\sqrt{2}$ b) $(2)^{1/3}$ c) $\sqrt{3}/2$ d) $2^{2/3}$

Soln

$S = (1+x)^{-p/q}$

$= (1+2/3)^{-1/2}$

$= (1+1/3)^{-1/2}$

$= (4/3)^{-1/2}$

$= (3/4)^{-1/2}$

$= \frac{\sqrt{3}}{2}$

Ans : c

6) $1 + \frac{2}{6} + \frac{2.5}{6.12} + \frac{2.5.8}{6.12.18} + \cdots$ = a) $\frac{\sqrt{3}}{2}$ b) $2^{1/3}$ c) $\frac{\sqrt{2}}{3}$ d) $2^{2/3}$

Soln

$S = (1-x)^{-p/q}$

$= (1-\frac{3}{6})^{-2/3}$

$= (1/2)^{-2/3}$

$= (2)^{2/3}$

Ans : d

7. $1-\frac{1}{4}+\frac{1\cdot3}{4\cdot8}-\frac{1\cdot3\cdot5}{4\cdot8\cdot12}\cdots =$ a) $\sqrt{2/3}$ b) $(\frac{2}{3})^{3/2}$ c) $\sqrt{3}$ d) $2^{2/3}$

Soln

$S = (1+x)^{-p/q}$

$= (1+\frac{2}{4})^{-1/2}$

$= (3/2)^{-1/2}$

$= \sqrt{2/3}$

Ans: a

8. $1+\frac{1}{3}+\frac{1\cdot3}{3\cdot6}+\frac{1\cdot3\cdot5}{3\cdot6\cdot9}+\cdots =$ a) $\sqrt{2/3}$ b) $(\frac{2}{3})^{3/2}$ c) $\sqrt{3}$ d) $2^{3/2}$

Soln

$S = (1-x)^{-p/q}$

$= (1-2/3)^{-1/2}$

$= (1/3)^{-1/2}$

$= \sqrt{3}$

Ans: c

9. $1-\frac{3}{4}+\frac{3\cdot5}{4\cdot8}-\frac{3\cdot5\cdot7}{4\cdot8\cdot12}+\cdots =$ a) $\sqrt{2/3}$ b) $(\frac{2}{3})^{3/2}$ c) $\sqrt{3}$ d) $2^{2/3}$

Soln

$S = (1+x)^{-p/q}$

$= (1+\frac{2}{4})^{-3/2}$

$= (3/2)^{-3/2}$

$= (2/3)^{3/2}$

Ans: b

10) $\frac{1}{4}-\frac{1\cdot3}{4\cdot8}+\frac{1\cdot3\cdot5}{4\cdot8\cdot12}\cdots =$ a) $1-\sqrt{\frac{2}{3}}$ b) $4^{1/3}-1$ c) $1-(5/8)^{1/3}$ d) $2^{1/3}-1$

Soln

$= -(1+x)^{-p/q}+1$

$= -[1+\frac{2}{4}]^{-1/2}+1$

$= -(3/2)^{-1/2} + 1$

$= 1 - (\frac{2}{3})^{1/2}$

Ans: a

11. $\frac{2}{6} + \frac{2.5}{6.12} + \frac{2.5.8}{4.8.12} + \ldots =$ a) $1-\sqrt{\frac{2}{3}}$ b) $4^{1/3}-1$ c) $1-(5/8)^{1/3}$ d) $2^{1/3}-1$

Soln

$= (1-x)^{-p/q} - 1$

$= (1-3/6)^{-2/3} - 1$

$= (1-1/2)^{-2/3} - 1$

$= (2)^{2/3} - 1$

$= (4)^{1/3} - 1$

Ans: b

12 $\frac{1}{6} + \frac{1.4}{6.12} + \frac{1.4.7}{6.12.18} + \ldots =$ a) $1-\sqrt{2/3}$ b) $4^{1/3}-1$ c) $1-(5/8)^{1/3}$ d) $2^{1/3}-1$

Soln

$= (1-x)^{-p/q} - 1$

$= (1-3/6)^{-1/3} - 1$

$= (1/2)^{-1/3} - 1$

$= 2^{1/3} - 1$

Ans: d

13. $\frac{1}{5} - \frac{1.4}{5.10} + \frac{1.4.7}{5.10.15} + \ldots =$ a) $1-\sqrt{\frac{2}{3}}$ b) $4^{1/3}-1$ c) $1-(\frac{5}{8})^{1/3}$ d) $2^{1/3}-1$

Soln

$= -(1+x)^{-p/q} + 1$

$= -(1+3/5)^{-1/3} + 1$

$= -(8/5)^{-1/3}$

14 $1-\frac{5}{6}(1/3)+\frac{5\cdot11}{6\cdot12}(1/3)^2+\cdots =$

a) $(3/4)^{5/4}$ b) x^n c) $\frac{1}{2^n}$ d) none.

Soln

$S = (1+x)^{-p/q}$

$= (1+1/3)^{-5/4}$

$= (4/3)^{-5/4}$

$= (3/4)^{5/4}$

Ans: a

Some important results:

1. $(1-x)^{-n} = 1+\frac{n}{1!}x+\frac{n(n+1)}{2!}x^2+\frac{n(n+1)(n+2)}{3!}x^3+\cdots$

2. $(1+x)^{-n} = 1-\frac{n}{1!}x+\frac{n(n+1)}{2!}x^2-\frac{n(n+1)(n+2)}{3!}x^3+\cdots$

3. $(1-x)^{-2} = 1+2x+3x^2+4x^3+\cdots$

problems:

1. $1+n\left(\frac{2x}{1+x}\right)+\frac{n(n+1)}{2!}\left(\frac{2x}{1+x}\right)^2+\cdots =$

a) $\left(\frac{1-x}{1+x}\right)^n$ b) $\left(\frac{1+x}{2x}\right)^n$ c) $\left(\frac{2x}{1+x}\right)^n$ d) $\left(\frac{1+x}{1-x}\right)^n$

Soln

$S = \left(1-\left(\frac{2x}{1+x}\right)\right)^{-n}$

$= \left(\frac{1-x}{1+x}\right)^{-n}$

$= \left(\frac{1+x}{1-x}\right)^{n}$

Ans: d

2. $1+n\left(1-\frac{1}{x}\right)+\frac{n(n+1)}{2!}\left(1-\frac{1}{x}\right)^2+\cdots =$

a) $(3/4)^{5/4}$ b) x^n c) $\frac{(1+x)^n}{2^n}$ d) $\left(\frac{1+x}{1-x}\right)^n$

$= \left(1-(1-\frac{1}{x})\right)^{-n}$

$= \left(\frac{1}{x}\right)^{-n}$

$= x^n$

Ans : b

3. $1 - n\left(\frac{1-x}{1+x}\right) + \frac{n(n+1)}{2!}\left(\frac{1+x}{1+x}\right)^2 - \cdots =$

a) $(3/4)^{5/4}$ b) x^n c) $\frac{(1+x)^n}{2^n}$ d) $\left(\frac{1+x}{1-x}\right)^n$

Soln

$S = (1+y)^{-n}$

$= \left(1+\left(\frac{1-x}{1+x}\right)\right)^{-n}$

$= \left(\frac{2}{1+x}\right)^n$

$= \frac{(1+x)^n}{2^n}$

Ans: c

4 $1 + 2\left(\frac{1}{2}\right) + 3\left(\frac{1}{2}\right)^2 + 4\left(\frac{1}{2}\right)^3 + \cdots =$ a) -1 b) 2 c) 4 d) 3

Soln

$S = (1-x)^{-1}$

$= (1-x)^{-2}$

$= (1-1/2)^{-2}$

$= (1/2)^{-2}$

$= 4$

Ans: c

5. $1 + (1/2) + (1/2)^2 + \cdots =$ a) 8 b) 2 c) 4 d) 3

Soln

$S = (1-x)^{-1}$

$= (1-1/2)^{-1}$

$= (1/2)^{-1}$

Binomial series

1.The expansion of $(3x-4)^{-7}$ is valid if

a)$|x|<4/3$ b) $|x|<1$ c) $|x|<4$ d) $\left|\frac{2x}{3}\right|<1$

2. The expansion of $(1-2x)^{-1}$ is valid if

a)$|x|<2$ b) $|x|<1$ c) $|x|<1/2$ d) $|x|<0$

3 .The expansion of $(1+3x)^{5/2}$ is valid if

a)$|x|<2$ b) $|x|<3$ c) $|x|<1/3$ d) $|x|<5$

4 .The coeff. of x^n in $(1-x)^{-1}$ is

a)0 b) 1 c) 1/n! d) n(n+1)

5.The coeff. of x^7 in $(1+2x)^{-1}$ is

a) 64 b)-64 c) 128 d) -128

6.The coeff. of x^n in $(1-3x)^{-1}$ is

a) 1 b)$3^n/n!$ c) 3^n d) $3^n(-1)^n$

7. $1+2\left(\frac{1}{2}\right)+3\left(\frac{1}{2}\right)^2+4\left(\frac{1}{2}\right)^3+\cdots=$

a) –1 b) 2 c) 4 d) 3

8. $1+\boldsymbol{n}\left(\frac{2x}{1+x}\right)+\frac{n,(n+1)}{2!}\left(\frac{2x}{1+x}\right)^2+\ldots=$

a) $\left(\frac{1-x}{1+x}\right)^n$ b) $\left(\frac{1+x}{2x}\right)^n$ c)$\left(\frac{2x}{1+x}\right)^n$ d) $\left(\frac{1+x}{1-x}\right)^n$

9. $\mathbf{1}-\frac{3}{4}+\frac{3.5}{4.8}-\frac{3.5.7}{4.8.12}\ldots=$

a) $\sqrt{\frac{2}{3}}$ b)$\left(\frac{2}{3}\right)^{\frac{3}{2}}$ c)$\sqrt{3}$ d)$2^{\frac{2}{3}}$

10 . $\mathbf{1}+\frac{3}{4}+\frac{3.5}{4.8}+\frac{3.5.7}{4.8.12}\ldots=$

a) $2\sqrt{2}$ b) $\sqrt[3]{2}$ c)$\sqrt{2}/3$ d)$2^{2/3}$

11. $\mathbf{1}-\frac{1}{5}+\frac{1.4}{5.10}+\frac{1.4.7}{5.10.15}\ldots=$

a) $2\sqrt{2}$ b) $\sqrt[3]{2}$ c)$\sqrt{2}/3$ d)$2^{2/3}$

12. $\mathbf{1}+\frac{\mathbf{1}}{\mathbf{6}}+\frac{\mathbf{1.4}}{\mathbf{6.12}}+\frac{\mathbf{1.4.7}}{\mathbf{6.12.18}}\ldots\ =$

a) $2\sqrt{2}$ b) $\sqrt[3]{2}$ c)$\sqrt{2}/3$ d)$2^{2/3}$

13. $\mathbf{1}-\frac{\mathbf{1}}{\mathbf{6}}+\frac{\mathbf{1.3}}{\mathbf{6.12}}-\frac{\mathbf{1.3.5}}{\mathbf{6.12.18}}\ldots\ =$

a) $\sqrt{3}/2$ b) $\sqrt[3]{2}$ c)$\sqrt{2}/3$ d)$2^{2/3}$

14. $\mathbf{1}+\frac{\mathbf{2}}{\mathbf{6}}+\frac{\mathbf{2.5}}{\mathbf{6.12}}+\frac{\mathbf{2.5.8}}{\mathbf{6.12.18}}\ldots\ =$

a) $\sqrt{2/3}$ b)$(\frac{2}{3})^{3/2}$ c)$\sqrt{3}$ d)$2^{2/3}$

15. $\mathbf{1}-\frac{\mathbf{1}}{\mathbf{4}}+\frac{\mathbf{1.3}}{\mathbf{4.8}}-\frac{\mathbf{1.3.5}}{\mathbf{4.8.12}}\ldots\ =$

a) $\sqrt{2/3}$ b)$(\frac{2}{3})^{3/2}$ c)$\sqrt{3}$ d)$2^{2/3}$

16. $\mathbf{1}+\frac{\mathbf{1}}{\mathbf{3}}+\frac{\mathbf{1.3}}{\mathbf{3.6}}+\frac{\mathbf{1.3.5}}{\mathbf{3.6.9}}\ldots\ =$

a) $\sqrt{2/3}$ b)$(\frac{2}{3})^{3/2}$ c)$\sqrt{3}$ d)$2^{2/3}$

17. $\mathbf{1}-\frac{\mathbf{3}}{\mathbf{4}}+\frac{\mathbf{3.5}}{\mathbf{4.8}}-\frac{\mathbf{3.5.7}}{\mathbf{4.8.12}}\ldots\ =$

a) $\sqrt{2/3}$ b)$(\frac{2}{3})^{3/2}$ c)$\sqrt{3}$ d)$2^{2/3}$

18. $\frac{\mathbf{1}}{\mathbf{4}}-\frac{\mathbf{1.3}}{\mathbf{4.8}}+\frac{\mathbf{1.3.5}}{\mathbf{4.8.12}}\ldots\ =$

a) $1-\sqrt{2/3}$ b)$4^{1/3}-1$ c)$1-(5/8)^{1/3}$ d)$2^{1/3}-1$

19. $\frac{\mathbf{2}}{\mathbf{6}}+\frac{\mathbf{2.5}}{\mathbf{6.12}}+\frac{\mathbf{2.5.8}}{\mathbf{4.8.12}}\ldots\ =$

a) $1-\sqrt{2/3}$ b)$4^{1/3}-1$ c)$1-(5/8)^{1/3}$ d)$2^{1/3}-1$

20. $\frac{\mathbf{1}}{\mathbf{6}}+\frac{\mathbf{1.4}}{\mathbf{6.12}}+\frac{\mathbf{1.4.7}}{\mathbf{6.12.18}}\ldots\ =$

a) $1-\sqrt{2/3}$ b)$4^{1/3}-1$ c)$1-(5/8)^{1/3}$ d)$2^{1/3}-1$

21 . $\frac{\mathbf{1}}{\mathbf{5}}-\frac{\mathbf{1.4}}{\mathbf{5.10}}+\frac{\mathbf{1.4.7}}{\mathbf{5.1.15}}\ldots\ =$

a) $1-\sqrt{2/3}$ b)$4^{1/3}-1$ c)$1-(5/8)^{1/3}$ d)$2^{1/3}-1$

22. $1-\frac{\mathbf{5}}{\mathbf{6}}\left(\frac{\mathbf{1}}{\mathbf{3}}\right)+\frac{\mathbf{5.11}}{\mathbf{6.12}}\left(\frac{\mathbf{1}}{\mathbf{3}}\right)^{\mathbf{2}}-\ldots\ =$

a) $(3/4)^{5/4}$ b)x^n c)$\frac{(1+x)^n}{2^n}$ d) $\left(\frac{1+x}{1-x}\right)^n$

23. $1+n\left(\mathbf{1}-\frac{\mathbf{1}}{x}\right)+\frac{n.(n+1)}{\mathbf{1.2}}\left(\mathbf{1}-\frac{\mathbf{1}}{x}\right)^{\mathbf{2}}+\ldots\ =$

a) $(3/4)^{5/4}$ b)x^n c)$\frac{(1+x)^n}{2^n}$ d) $\left(\frac{1+x}{1-x}\right)^n$

24. $1 - n\left(\frac{1-x}{1+x}\right) + \frac{n,(n+1)}{2!}\left(\frac{1-x}{1+x}\right)^2 - \ldots =$

a) $(3/4)^{5/4}$ b)x^n c)$\frac{(1+x)^n}{2^n}$ d) $\left(\frac{1+x}{1-x}\right)^n$

25. $1 + n\left(\frac{2x}{1+x}\right) + \frac{n,(n+1)}{2!}\left(\frac{2x}{1+x}\right)^2 + \ldots =$

a) $(3/4)^{5/4}$ b)x^n c)$\frac{(1+x)^n}{2^n}$ d) $\left(\frac{1+x}{1-x}\right)^n$

Answer key :

1	6	11	16	21
2	7	12	17	22
3	8	13	18	23
4	9	14	19	24
5	10	15	20	25