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Electromagnetic theory:  

Electromagnetic theory is concerned with the study of charges at rest and in motion. Electromagnetic principles are fundamental to the study of electrical engineering. Electromagnetic theory is also required for the understanding, analysis and design of various electrical, electromechanical and electronic systems.

Electromagnetic theory can be thought of as generalization of circuit theory. Electromagnetic theory deals directly with the electric and magnetic field vectors whereas circuit theory deals with the voltages and currents. Voltages and currents are integrated effects of electric and magnetic fields respectively.

Electromagnetic field problems involve three space variables along with the time variable and hence the solution tends to become correspondingly complex.

Vector analysis is the required mathematical tool with which electromagnetic concepts can be conveniently expressed and best comprehended. Since use of vector analysis in the study of electromagnetic field theory is prerequisite, first we will go through vector algebra.
Applications of Electromagnetic theory:

This subject basically consist of static electric fields, static magnetic fields, time-varying fields & it’s applications.

One of the most common applications of electrostatic fields is the deflection of a charged particle such as an electron or proton in order to control it’s trajectory.

The deflection is achieved by maintaining a potential difference between a pair of parallel plates.

This principle is used in CROs, ink-jet printer etc. Electrostatic fields are also used for sorting of minerals for example in ore separation. Other applications are in electrostatic generator and electrostatic voltmeter.

The most common applications of static magnetic fields are in dc machines. Other applications include magnetic deflection, magnetic separator, cyclotron, hall effect sensors, magnetohydrodynamic generator etc.

Vector Analysis:

The quantities that we deal in electromagnetic theory may be either scalar or vectors. Scalars are quantities characterized by magnitude only. A quantity that has direction as well as magnitude is called a vector.

In electromagnetic theory both scalar and vector quantities are function of time and position.

A vector \( \vec{A} \) can be written as, \( \vec{A} = \hat{A} \hat{A} \), where, \( \hat{A} = \frac{\vec{A}}{|\vec{A}|} \) is the magnitude and is the unit vector which has unit magnitude and same direction as that of \( \vec{A} \).

Two vector \( \vec{A} \) and \( \vec{B} \) are added together to give another vector \( \vec{C} \). We have

\[ \vec{C} = \vec{A} + \vec{B} \]

Let us see the animations in the next pages for the addition of two vectors, which has two rules:

1: Parallelogram law and 2: Head & tail rule
Scaling of a vector is defined as where is scaled version of vector and is a scalar.

Some important laws of vector algebra are:

\[ \vec{A} + \vec{B} = \vec{B} + \vec{A} \]  
Commutative Law

\[ \vec{A} + (\vec{B} + \vec{C}) = (\vec{A} + \vec{B}) + \vec{C} \]  
Associative Law

\[ \alpha(\vec{A} + \vec{B}) = \alpha\vec{A} + \alpha\vec{B} \]  
Distributive Law

The position vector \( \vec{r}_O \) of a point \( P \) is the directed distance from the origin \((O)\) to \( P \), i.e., \( \vec{r}_O = \overrightarrow{OP} \).

If \( \vec{r}_P = \overrightarrow{OP} \) and \( \vec{r}_Q = \overrightarrow{OQ} \) are the position vectors of the points \( P \) and \( Q \) then the distance vector \( \vec{PQ} = \overrightarrow{OQ} - \overrightarrow{OP} = \vec{r}_Q - \vec{r}_P \).

**Product of Vectors**

When two vectors \( \vec{A} \) and \( \vec{B} \) are multiplied, the result is either a scalar or a vector depending how the two vectors were multiplied. The two types of vector multiplication are:

- Scalar product (or dot product) \( \vec{A} \cdot \vec{B} \) gives a scalar.
- Vector product (or cross product) \( \vec{A} \times \vec{B} \) gives a vector.

The dot product between two vectors is defined as

\[ \vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos\theta_{AB} \]

Where \( \vec{n} \) is unit vector perpendicular to \( \vec{A} \) and \( \vec{B} \)

Vector dot product
The dot product is commutative i.e., \( \vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A} \) and distributive
\( \vec{A} \cdot (\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C} \).

Associative law does not apply to scalar product.

The vector or cross product of two vectors \( \vec{A} \) and \( \vec{B} \) is denoted by \( \vec{A} \times \vec{B} \). \( \vec{A} \times \vec{B} \) is a vector perpendicular to the plane containing \( \vec{A} \) and \( \vec{B} \), the magnitude is given by \( |\vec{A}||\vec{B}| \sin \theta_{AB} \) and direction is given by right hand rule.

\[ \hat{a}_n = \frac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|} \]

The following relations hold for vector product:

\[ \vec{A} \times \vec{B} = -\vec{B} \times \vec{A} \] i.e., cross product is non commutative

\[ \vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C} \] i.e., cross product is distributive

\[ \vec{A} \times (\vec{B} \times \vec{C}) \neq (\vec{A} \times \vec{B}) \times \vec{C} \] i.e., cross product is non associative

Scalar and vector triple product:

Scalar triple product
\[ \vec{A} \cdot (\vec{B} \times \vec{C}) = \vec{B} \cdot (\vec{C} \times \vec{A}) = \vec{C} \cdot (\vec{A} \times \vec{B}) \]

Vector triple product
\[ \vec{A} \times (\vec{B} \times \vec{C}) = \vec{B} (\vec{A} \cdot \vec{C}) - \vec{C} (\vec{A} \cdot \vec{B}) \]

**Co-ordinate Systems:**

In order to describe the spatial variations of the quantities, we require using appropriate co-ordinate system. A point or vector can be represented in an orthogonal coordinate system.

An orthogonal system is one in which the co-ordinates are mutually
perpendicular.

In electromagnetic theory many physical quantities are vectors, which are having different components.

So we use orthogonal co-ordinate systems for representing those quantities and depending on the symmetry of the physical quantities different coordinate systems are used.
Cartesian Co-ordinate System:

A point \( P(x, y, z) \) in Cartesian co-ordinate system is represented as intersection of three planes \( x = \text{constant}, y = \text{constant} \) and \( z = \text{constant} \), as shown in the figure below. The unit vectors along the three axes are as shown in the figure.

Coordinate system represented by \((x, y, z)\) that are three orthogonal vectors in straight lines that intersect at a single point (the origin). The range of variation along the three axes are shown below.

\[-\infty < x < \infty, -\infty < y < \infty, -\infty < z < \infty\]

The vector \( A \) in this coordinate system can be written as, \( \vec{A} = A_x \hat{a}_x + A_y \hat{a}_y + A_z \hat{a}_z \)

The differential lengths, area and volumes are as shown below.
**Cylindrical Co-ordinate System:**

For cylindrical coordinate systems we have \((\alpha, \nu, \omega) = (r, \phi, z)\) as shown in figure below.

Cylindrical Coordinate System

Cylindrical Coordinate system represented by \((r, \phi, z)\) that are three orthogonal vectors, varies in the range,

\[
0 \leq \rho < \infty \\
0 \leq \phi < 2\pi \\
-\infty < z < \infty
\]

The vector \(A\) in this coordinate system can be written as,

\[
\vec{A} = A_\rho \hat{\rho} + A_\phi \hat{\phi} + A_z \hat{z}
\]

The following equations can be used to convert between cylindrical and Cartesian coordinate systems,

\[
\rho = \sqrt{x^2 + y^2} \\
\phi = \tan^{-1}\left(\frac{y}{x}\right) \\
x = \rho \cos \phi \\
y = \rho \sin \phi \\
z = z
\]

The differential elements in cylindrical coordinate system are shown below.
Spherical co-ordinate system:

Coordinate system represented by \((r, \theta, \phi)\) that are three orthogonal vectors (as shown in the figure below) emanating from or revolving around the origin in the range,

\[
0 \leq r < \infty \\
0 \leq \theta \leq \pi \\
0 \leq \phi \leq 2\pi
\]

The unit vectors in the three orthogonal directions are, \(\hat{z}\).

The vector \(\mathbf{A}\) in this coordinate system can be written as,

\[
\mathbf{A} = A_r \hat{r} + A_{\theta} \hat{\theta} + A_{\phi} \hat{\phi}
\]

The following equations can be used to convert between spherical and Cartesian coordinate systems.

\[
\begin{align*}
  r &= \sqrt{x^2 + y^2 + z^2} \\
  \theta &= \tan^{-1} \left( \frac{\sqrt{x^2 + y^2}}{z} \right) \\
  \phi &= \tan^{-1} \left( \frac{y}{x} \right) \\
  x &= r \sin \theta \cos \phi \\
  y &= r \sin \theta \sin \phi \\
  z &= r \cos \theta
\end{align*}
\]
The differential elements in spherical coordinate system are shown below.

\[ dl^2 = dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \]

\[ dS^2 = r^2 \sin \theta d\theta d\phi \]

\[ dv = r^2 \sin \theta dr d\theta d\phi \]

Co-ordinate transformation:

Matrix Transformations: Cartesian to Cylindrical

\[
\begin{bmatrix}
A_r \\
A_\phi \\
A_z
\end{bmatrix} =
\begin{bmatrix}
\cos \phi & \sin \phi & 0 \\
-\sin \phi & \cos \phi & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
A_x \\
A_y \\
A_z
\end{bmatrix}
\]

\[
\begin{bmatrix}
A_x \\
A_y \\
A_z
\end{bmatrix} =
\begin{bmatrix}
\cos \phi & -\sin \phi & 0 \\
\sin \phi & \cos \phi & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
A_r \\
A_\phi \\
A_z
\end{bmatrix}
\]

Matrix Transformations: Cartesian to Spherical

\[
\begin{bmatrix}
A_r \\
A_\theta \\
A_\phi
\end{bmatrix} =
\begin{bmatrix}
\sin \theta \cos \phi & \sin \theta \sin \phi & \cos \theta \\
\cos \theta \cos \phi & \cos \theta \sin \phi & -\sin \theta \\
-\sin \theta & \cos \phi & 0 \\
\end{bmatrix}
\begin{bmatrix}
A_x \\
A_y \\
A_z
\end{bmatrix}
\]

\[
\begin{bmatrix}
A_x \\
A_y \\
A_z
\end{bmatrix} =
\begin{bmatrix}
\sin \theta \cos \phi & \cos \theta \cos \phi & -\sin \theta \\
\cos \theta \sin \phi & \cos \theta \sin \phi & \cos \theta \\
\cos \phi & \sin \phi & 0 \\
\end{bmatrix}
\begin{bmatrix}
A_r \\
A_\theta \\
A_\phi
\end{bmatrix}
\]
Del operator:

Del is a vector differential operator. The del operator will be used in for differential operations throughout any course on field theory. The following equation is the del operator for different coordinate systems.

\[
\nabla = \frac{\partial}{\partial x} \hat{a}_x + \frac{\partial}{\partial y} \hat{a}_y + \frac{\partial}{\partial z} \hat{a}_z = \nabla_{xyz} \\
\n\nabla = \frac{1}{\rho} \frac{\partial}{\partial \rho} \hat{a}_\rho + \frac{\partial}{\partial \phi} \hat{a}_\phi + \frac{\partial}{\partial z} \hat{a}_z \\
\n\nabla = \frac{1}{r} \frac{\partial}{\partial r} \hat{a}_r + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} \hat{a}_\theta + \frac{1}{r \sin \theta} \frac{\partial}{\partial \phi} \hat{a}_\phi
\]

Gradient of a Scalar:

• The gradient of a scalar field, V, is a vector that represents both the magnitude and the direction of the maximum space rate of increase of V.

\[
\nabla V = \frac{\partial V}{\partial x} \hat{a}_x + \frac{\partial V}{\partial y} \hat{a}_y + \frac{\partial V}{\partial z} \hat{a}_z = \nabla V_{xyz}
\]

• To help visualize this concept, take for example a topographical map. Lines on the map represent equal magnitudes of the scalar field.

• The gradient vector crosses map at the location where the lines packed into the most dense space and perpendicular (or normal) to them.

• The orientation (up or down) of the gradient vector is such that the field is increased in magnitude along that direction.

Fundamental properties of the gradient of a scalar field

* The magnitude of gradient equals the maximum rate of change in V per unit distance
* Gradient points in the direction of the maximum rate of change in V
* Gradient at any point is perpendicular to the constant V surface that passes through that point
* The projection of the gradient in the direction of the unit vector \( \mathbf{a} \), is \( \nabla V \cdot \hat{a} \) and is called the directional derivative of V along \( \mathbf{a} \).
This is the rate of change of $V$ in the direction of $\mathbf{a}$.
* If $\mathbf{A}$ is the gradient of $V$, then $V$ is said to be the scalar potential of $\mathbf{A}$.

\[
\vec{A} = A_1\hat{a}_1 + A_2\hat{a}_2 + A_3\hat{a}_3
\]

\[
div \vec{A} = \nabla \cdot \vec{A} = \lim_{\Delta v \to 0} \frac{\int_S \vec{A} \cdot d\vec{S}}{\Delta v}
\]

**Divergence of a Vector:**
- The divergence of a vector, $\mathbf{A}$, at any given point $P$ is the outward flux per unit volume as volume shrinks about $P$.

\[
div \vec{A} = \nabla \cdot \vec{A} = \lim_{\Delta v \to 0} \frac{\int_S \vec{A} \cdot d\vec{S}}{\Delta v}
\]

**Divergence Theorem:**
- The divergence theorem states that the total outward flux of a vector field, $\mathbf{A}$, through the closed surface, $S$, is the same as the volume integral of the divergence of $\mathbf{A}$.
- This theorem is easily shown from the equation for the divergence of a vector field.

**Curl of a Vector:**
- The curl of a vector, $\mathbf{A}$ is an axial vector whose magnitude is the maximum circulation of $\mathbf{A}$ per unit area as the area tends to zero and whose direction is the normal direction of the area when the area is oriented to make the circulation maximum.

**Stokes Theorem:**
- Stokes theorem states that the circulation of a vector field $\mathbf{A}$, around a closed path, $L$ is equal to the surface integral of the curl of $\mathbf{A}$ over the open surface $S$ bounded by $L$.
- This theorem has been proven to hold as long as $\mathbf{A}$ and the curl of $\mathbf{A}$ are continuous along the closed surface $S$ of a closed path $L$.
- This theorem is easily shown from the equation for the curl of a vector field.
Classification of vector field:

The vector field, \( \mathbf{A} \), is said to be divergenceless (or solenoidal) if \( \nabla \cdot \mathbf{A} = 0 \). Such fields have no source or sink of flux, thus all the vector field lines entering an enclosed surface, \( S \), must also leave it.

Examples include magnetic fields, conduction current density under steady state, and incompressible fluids.

The following equations are commonly utilized to solve divergenceless field problems:

\[
\nabla \cdot \mathbf{A} = 0 \\
\oint_S \mathbf{A} \cdot d\mathbf{S} = \int (\nabla \times \mathbf{A}) \cdot d\mathbf{V} = 0 \\
\n\mathbf{F} = \nabla \times \mathbf{A}
\]

1. Given the vectors:

\[
\mathbf{A} = \hat{a}_1 + 2\hat{a}_2 + 3\hat{a}_3 \\
\mathbf{B} = 5\hat{a}_r - \hat{a}_p + 2\hat{a}_z
\]

Find:

a. The vector \( \mathbf{C} = \mathbf{A} + \mathbf{B} \) at a point \( P(0, 2, -3) \).

b. The component of \( \mathbf{A} \) along \( \mathbf{B} \) at \( P \).

Solution:

The vector \( \mathbf{B} \) is cylindrical coordinates. This vector in Cartesian coordinate can be written as:

\[ \mathbf{B} = B_x \hat{a}_x + B_y \hat{a}_y + B_z \hat{a}_z \]

Where

\[ B_x = \mathbf{B} \cdot \hat{a}_x = 5\hat{a}_r \cdot \hat{a}_x - \hat{a}_p \cdot \hat{a}_x + 3\hat{a}_z \cdot \hat{a}_x = 5 \cos \phi + \sin \phi \]

\[ B_y = \mathbf{B} \cdot \hat{a}_y = 5 \sin \phi - \cos \phi \]
The point P(0,2,-3) is in the y-z plane for which .

\[ B = \hat{a}_x + 5\hat{a}_y + 3\hat{a}_z = \frac{\pi}{2} \]

\[ a_{12} = \frac{\vec{r}_2 - \vec{r}_1}{R} \]

\[ a_{21} = \frac{\vec{r}_1 - \vec{r}_2}{R} \]

a. For evaluating the line integral along the parabola \( y = x^2 \), we find that \( dy = 2x \, dx \)

\[ \int_{a}^{b} F \cdot dl = \int_{a}^{b} x^2 \, dx + 2x^2 \, dx \]

\[ = \int_{a}^{b} 3x^2 \, dx = \left[ x^3 \right]_a^b = z \]

Field:

A field is a function that specifies a particular physical quantity everywhere in a region. Depending upon the nature of the quantity under consideration, the field may be a vector or a scalar field. Example of scalar field is the electrostatic potential in a region while electric or magnetic fields at any point is the example of vector field.

**Static Electric Fields:**

Electrostatics can be defined as the study of electric charges at rest. Electric fields have their sources in electric charges. The fundamental & experimentally proved laws of electrostatics are Coulomb’s law & Gauss’s theorem.

**Coulomb’s law & Electric field Intensity:**

**Statement:** force between two point charges separated in vacuum or free space by a distance which is large compared to their size is directly proportional to the product of their charges and inversely proportional to the square of the distance between them. It acts along the line joining the two charges. Mathematically,

\[ F = \frac{kQ_1Q_2}{R^2} \]

In SI units, \( Q_1 \) and \( Q_2 \) are expressed in Coulombs(C) and \( R \) is in meters.

\[ k = \frac{1}{4\pi \varepsilon_0} \]

Force \( F \) is in Newtons (N) and \( \varepsilon_0 \) is called the permittivity of free space & it’s magnitude is

\[ \varepsilon_0 = 8.854 \times 10^{-12} = \frac{1}{36} \times 10^9 \text{ F/m} \]

Therefore
Let the position vectors of the point charges \( Q_1 \) and \( Q_2 \) are given by \( \vec{r}_1 \) and \( \vec{r}_2 \). Let \( \vec{R}_{12} \) represent the force on \( Q_1 \) due to charge \( Q_2 \).

The charges are separated by a distance of \( \vec{R} = |\vec{r}_1 - \vec{r}_2| = |\vec{r}_2 - \vec{r}_1| \). We define the unit vectors as

\[
\vec{R}_{12} = \frac{Q_1 Q_2}{4 \pi \varepsilon_0 |\vec{R}|^2} \frac{\vec{r}_2 - \vec{r}_1}{|\vec{r}_2 - \vec{r}_1|}
\]

\( \vec{R}_{12} \) can be defined as

Similarly, the force on \( Q_1 \) due to charge \( Q_2 \) can be calculated and if \( \vec{F}_{21} \) represents this force then we can write \( \vec{F}_{21} = -\vec{F}_{12} \).

Suppose a charge \( q \) is placed in the vicinity of three other charges, \( q_1, q_2, \) and \( q_3 \). Coulomb's law can be used to calculate the electric force between \( q \) and \( q_1 \), between \( q \) and \( q_2 \), and between \( q \) and \( q_3 \). Experiments have shown that the total force exerted by \( q_1, q_2 \) and \( q_3 \) on \( q \) is the vector sum of the individual forces, as shown in the equation below:

\[
\vec{E}(\vec{r}) = \frac{q_1}{4 \pi \varepsilon_0 |\vec{r} - \vec{r}_1|^3} \vec{r} - \vec{r}_1 + \frac{q_2}{4 \pi \varepsilon_0 |\vec{r} - \vec{r}_2|^3} \vec{r} - \vec{r}_2 + \frac{q_3}{4 \pi \varepsilon_0 |\vec{r} - \vec{r}_3|^3} \vec{r} - \vec{r}_3
\]

**Electric Field:**

The electric field intensity or the electric field strength at a point is defined as the force per unit charge. That is

\[
\vec{E} = \lim_{Q \to 0} \frac{\vec{F}}{Q} \quad \text{or,} \quad \vec{E} = \frac{\vec{F}}{Q}
\]

The electric field intensity \( E \) at a point \( r \) (observation point) due a point charge \( Q \) located at \( \vec{r}' \) (source point) is given by:

\[
\vec{E} = \frac{Q (r - r')} {4 \pi \varepsilon_0 |\vec{r} - \vec{r}'|} \]
For a collection of $N$ point charges $Q_1, Q_2, \ldots, Q_N$ located at the electric field intensity at point $i$ is obtained as

$$E_i = \frac{1}{4\pi \varepsilon_0} \sum_{j=1}^{N} \frac{Q_j}{|r_i - r_j|}$$

The expression can be modified suitably to compute the electric field due to a continuous distribution of charges.

For an elementary charge $dQ = \rho(r')dv'$, i.e. considering this charge as point charge, we can write the field expression as:

$$dE = \frac{dQ}{4\pi \varepsilon_0 |r - r'|} = \frac{\rho(r')dv'(r - r')}{4\pi \varepsilon_0 |r - r'|^3}$$

When this expression is integrated over the source region, we get the electric field at the point $P$ due to this distribution of charges. Thus the expression for the electric field at $P$ can be written as:

$$E(r) = \frac{\int \rho(r')(r - r')}{4\pi \varepsilon_0 |r - r'|^3}$$

Similar technique can be adopted when the charge distribution is in the form of a line charge density or a surface charge density.

Electric flux density:

As stated earlier electric field intensity or simply ‘Electric field’ gives the strength of the field at a particular point. The electric field depends on the material media in which the field is being considered.

The flux density vector is defined to be independent of the material media (as we'll see that it relates to the charge that is producing it). For a linear isotropic medium under consideration; the flux density vector is defined as: $\overrightarrow{D} = \varepsilon \overrightarrow{E}$

We define the electric flux $\psi$ as $\psi = \oint \overrightarrow{D} \cdot d\overrightarrow{s}$
Gauss's Law:

Gauss's law is one of the fundamental laws of electromagnetism and it states that the total electric flux through a closed surface is equal to the total charge enclosed by the surface.

![Gauss's Law](image)

**Application of Gauss's Law**

Gauss's law is particularly useful in computing $\vec{E}$ or $\vec{D}$ where the charge distribution has some symmetry.

We shall illustrate the application of Gauss's Law with some examples.

1. **An infinite line charge**

   Let’s consider the problem of determination of the electric field produced by an infinite line charge of density $\rho_L \text{C/m.}$ Let us consider a line charge positioned along the $z$-axis. Since the line charge is assumed to be infinitely long, the electric field will be of the form $E = \frac{\rho_L}{2\pi\epsilon_0} \hat{z}.$

   If we consider a close cylindrical surface, using Gauss's theorem we can write,

   $$\rho_L' = \oint_{S} \vec{E} \cdot d\vec{S} = \oint_{S_1} \vec{E} \cdot d\vec{S} + \oint_{S_2} \vec{E} \cdot d\vec{S} + \oint_{S_3} \vec{E} \cdot d\vec{S}$$

   Considering the fact that the unit normal vector to areas $S_1$ and $S_3$ are perpendicular to the electric field, the surface integrals for the top and bottom surfaces evaluates to zero.

   Hence we can write,

   $$\rho_L' = \oint_{S_2} \vec{E} \cdot d\vec{S} = \frac{\rho_L}{2\pi\epsilon_0} \hat{z}$$

2. **Infinite Sheet of Charge**

   As a second example of application of Gauss's theorem, we consider an infinite charged sheet covering the $x-z$ plane.

   Assuming a surface charge density of $\sigma_s$ for the infinite surface charge, if we consider a cylindrical volume having sides $\Delta s$ placed symmetrically.

   $$\oint_{\Delta s} \vec{D} \cdot d\vec{S} = 2\Delta s \sigma_s = \rho \Delta s$$

   $$\therefore \vec{E} = \frac{\sigma_s}{2\epsilon_0} \hat{y}$$

   It may be noted that the electric field strength is independent of distance. This is
true for the infinite plane of charge; electric lines of force on either side of the charge will be perpendicular to the sheet and extend to infinity as parallel lines.

As number of lines of force per unit area gives the strength of the field, the field becomes independent of distance. For a finite charge sheet, the field will be a function of distance.

\[
V(\mathbf{r}) = \frac{1}{4\pi \varepsilon_0} \left( \frac{Q_1}{|\mathbf{r} - \mathbf{r}_1|} + \frac{Q_2}{|\mathbf{r} - \mathbf{r}_2|} + \cdots + \frac{Q_n}{|\mathbf{r} - \mathbf{r}_n|} \right)
\]

**Electrostatic Potential and Equipotential Surfaces:**

Electrostatic potential is related to the work done in carrying a charge from one point to the other in the presence of an electric field. Let us suppose that we wish to move a positive test charge \( \Delta q \) from a point \( P \) to another point \( Q \).

The work done by this external agent in moving the charge by a distance \( d\mathbf{\ell} \) is given by:

\[
dW = -\Delta q \mathbf{E} \cdot d\mathbf{\ell}
\]

The negative sign accounts for the fact that work is done on the system by the external agent.

\[
W = -\Delta q \int_\mathbf{P}^{\mathbf{Q}} \mathbf{E} \cdot d\mathbf{\ell}
\]

The potential difference between two points \( P \) and \( Q \), \( V_{PQ} \), is defined as the work done per unit charge, i.e.

\[
V_{PQ} = \frac{W}{\Delta q} = -\int_\mathbf{P}^{\mathbf{Q}} \mathbf{E} \cdot d\mathbf{\ell}
\]

It may be noted that in moving a charge from the initial point to the final point if the potential difference is positive, there is a gain in potential energy in the movement, external agent performs the work against the field. If the sign of the potential difference is negative, work is done by the field.

We will see that the electrostatic system is conservative in that no net energy is exchanged if the test charge is moved about a closed path, i.e. returning to its initial position. Further, the potential difference between two points in an electrostatic field is a point function,
it is independent of the path taken. The potential difference is measured in Joules/Coulomb which is referred to as Volts.

Considering the movement of a unit positive test charge from an arbitrary point \( B \) to another arbitrary point \( A \), we can write an expression for the potential difference as:

\[
V_{Ba} = -\int_{B}^{A} \vec{E} \cdot d\vec{l} = -\int_{B}^{A} \frac{Q}{4\pi\varepsilon_{0}r^{2}} \hat{r} \cdot d\hat{r} = \frac{Q}{4\pi\varepsilon_{0}} \left[ \frac{1}{r_{A}} - \frac{1}{r_{B}} \right] = V_{A} - V_{B}
\]

So, the potential difference is independent of the path taken as it only depends on the initial & final points. It is customary to choose the potential to be zero at infinity.

Thus potential at any point \( (r_{A} = r) \) due to a point charge \( Q \) can be written as the amount of work done in bringing a unit positive charge from infinity to that point (i.e. \( r_{B} = 0 \)).

\[
V = \frac{1}{4\pi\varepsilon_{0}} \frac{Q}{r}
\]

Or, in other words,

\[
V = -\int_{\infty}^{r} \vec{E} \cdot d\vec{r}
\]

Let us now consider a situation where the point charge \( Q \) is not located at the origin.

The potential at a point \( P \) becomes:

\[
V(r) = \frac{Q}{4\pi\varepsilon_{0}} \frac{1}{r - r'}
\]

\[
V(r) = \frac{1}{4\pi\varepsilon_{0}} \sum_{n} \frac{Q_{n}}{r - r_{n}}
\]

For continuous charge distribution, we replace point charges \( Q_{n} \) by corresponding charge elements \( \rho_{x}dx \) or \( \rho_{y}dy \) or \( \rho_{z}dz \) depending on whether the charge distribution is linear, surface or a volume charge distribution and the summation is replaced by an integral. With these modifications we can write:

\[
V(r) = \frac{1}{4\pi\varepsilon_{0}} \int \frac{\rho_{x}dx}{r - r_{n}}
\]
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