NATIONAL ACADEMY DHARMAPURI

PG TRB MATHS MATERIALS & QUESTION PAPERS AVAILABLE...

CONTACT: 8248617507, 7010865319
1. The function \( \omega = \frac{z^2 + 1}{z^2 - 1} \) is analytic

(a) For all \( z \)                                    (b) For all \( z \) except (1,0),(-1,0)

(c) For all \( z \) except (0,1),(0,-1)                    (d) For all \( z \) except (0,1),(1,2)

2. If \( f(z) \) and \( f(\bar{z}) \) are analytic, then

(a) \( f(z) \) is constant    (b) \( f(z) \) is not constant   (c) \( f(z) \) is not analytic (d) \(|f(z)| \) is not constant

3. The Cartesian form of C.R equation is

(a) \( u_x = -v_y, u_y = v_x \)    (b) \( u_r = v_{\theta}, u_{\theta} = -v_r \)    (c) \( u_x = v_y, u_y = -v_x \)  (d) \( u_y = -v_y, u_y = v_x \)

4. If \( f(z) = \begin{cases} \frac{Re(z)}{|z|}, & \text{at } z \neq 0 \\ 0, & \text{at } z = 0 \end{cases} \)

Then at \( z = 0 \) is

(a) differentiable at \( z = 0 \)    (b) continuous at \( z = 0 \)

(c) not differentiable at \( z = 0 \)   (d) not differentiable and not continuous at \( z = 0 \)

5. If \( u = y^3 - 3x^2y \) then \( f(z) \) is

(a) \( f(z) = iz^3 + c \)    (b) \( -iz^3 + c \)  (c) \( z^3 + c \)  (d) \( iz^3 + 1 \)

6. \( u = e^x \sin y \) is harmonic, the value of \( v \) is

(a) \(-e^x \cos y + c\)    (b) \(-2e^x \cos y + c\)  (c) \(2e^x \cos y + c\)  (d) \(-2e^x \sin y\)

7. Let \( R(z) \) be a rational function of order 6, then \( R(z) \) has

(a) 3 zeros, 6 poles    (b) 12 zeros, 12 poles  (c) 6 zeros, 3 poles  (d) 6 zeros, 6 poles

8. If \( \lim_{n \to \infty} Z_n = A \), then \( \lim_{n \to \infty} \frac{1}{n}(Z1 + Z2 + \cdots + Zn) \) is

(a) \( \frac{1}{A} \)    (b) \( A \)  (c) \( \frac{1}{n} \)  (d) \( \frac{1}{n} A \)

9. If all zeros of a polynomial \( p(z) \) lie in the lower half-plane, then all zeros of \( p^1(z) \) lies in

(a) complex plane    (b) upper half plane  (c) lower half plane  (d) real plane
10. Which one of the following is not a bilinear transformation

(a) $3z+4$           (b) $\frac{3z+4}{5z+6}$           (c) $\frac{z+1}{z-1}$           (d) $\frac{z+3}{2z+6}$

11. The radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{n!}{2^n} z^n$ is

(a) $e$           (b) $\frac{1}{e}$           (c) $0$           (d) $\infty$

12. The radius convergence of $\sum_{n=1}^{\infty} \frac{2^n}{n!} z^n$ is

(a) $R = 0$           (b) $R = \infty$           (c) $R = e$           (d) $R = \frac{1}{e}$

13. Conformal mapping preserves

(a) direction only           (b) magnitude only           (c) magnitude and direction           (d) None of these

14. If $w = k z$ and $|k| = 1$, then $w$ is called

(a) inversion           (b) Rotation           (c) parallel translation           (d) homothetic

15. The cross ratio $(z_1, z_2, z_3, z_4)$ is real if and only if the four points are

(a) lie on a circle only           (b) lie on a circle or straight line           (c) lie on a straight line only           (d) lie on a sphere

16. The fixed point of $w = z + 3$ is

(a) $\infty$           (b) $0$           (c) undefined           (d) $-3$

17. An arc $z = z(t)$ is rectifiable $\iff$ the real and imaginary parts of $z(t)$ are

(a) not bounded           (b) equal           (c) not equal           (d) bounded variation

18. Which one of the following mapping is conformal

(a) $w = z^4$           (b) $w = z + \frac{1}{z}$           (c) $w = e^z$           (d) None

19. Which one of the following is correct

(a) $\int_{\gamma} \frac{dz}{z-a}$ is a multiple of $3\pi i$

(b) If $\gamma$ lies inside a circle, then $n(\gamma, a) = 0$ where $a$ is outside of the circle

(c) winding no is not an integer           (d) ‘a’ lies inside the circle and $\gamma$ lies outside the circle $n(\gamma, a) = 0$

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20. Let $n(\gamma, a)$ be a winding number of $\gamma$ with respect to ‘a’, then $n(\gamma, a)$
   (a) $n(\gamma, a)$  (b) $-n(-\gamma, a)$  (c) $n(-\gamma, a)$  (d) 0

21. If $\lim_{z\to a} f(z) = \infty$, then $a$ is
   (a) zero  (b) essential singularity  (c) pole  (d) Isolated E.S

22. A function which is analytic and bounded in the whole plane, then
   (a) it is a constant  (b) not constant  (c) purely real  (d) purely imaginary

23. The value of $\int_{|z|=2} \frac{dz}{z-1}$ is
   (a) 0  (b) $\pi$  (c) $2\pi i$  (d) $-2\pi i$

24. $f(z)$ is analytic in $\Omega$ and $f(z)$ is analytic then $|f(z)|$ has
   (a) max on the boundary only  (b) max inside the region $\Omega$ only
   (c) max inside and on the boundary  (d) None

25. residue of $f(z) = \frac{1}{z^2+1}$ at $z=i$ is
   (a) $2i$  (b) $-2i$  (c) $\frac{1}{2i}$  (d) $\frac{1}{2i}$

26. The Maclaurin's series of $e^{2z}$ is
   (a) $\sum_{n=0}^{\infty} \frac{z^n}{2n!}$  (b) $\sum_{n=0}^{\infty} \frac{(2z)^n}{n!}$  (c) $\sum_{n=1}^{\infty} \frac{(2z)^n}{n!}$  (d) $\sum_{n=1}^{\infty} \frac{(z/2)^n}{2n!}$

27. Morera’s theorem is the converse of
   (a) Cauchy Goursat theorem  (b) Lioville’s theorem
   (c) Cauchy integral theorem  (d) Fundamental theorem of algebra

28. If $f(z)$ is analytic with in and on $C$ and $z=a$ lies inside $C$, then $\int_C \frac{f(z)}{z-a} \, dz$ is
   (a) 0  (b) $2\pi i \times f(a)$  (c) $2\pi i f'(a)$  (d) $2\pi i$

29. Let $f(z) = \frac{1}{\sinhz}$ be a meromorphic in the complex plane. The limit point of the pole of this function is
   (a) $\pi$  (b) $0$  (c) $1$  (d) $\infty$

30. The function $e^{z}, \sin z$ and $\cos z$ have common essential singularity
   (a) $\infty$  (b) $0$  (c) $1$  (d) $\pi$

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31. The cross ratio of 1,2,3,4 is
(a) $-\frac{1}{3}$  (b) $\frac{1}{3}$  (c) 0  (d) $\infty$

32. The number of zero of $f(z) = z^4 - 5z + 1$ in $1 < |z| < 2$ is
(a) 3  (b) 2  (c) 1  (d) 4

33. If $f(z) = \frac{z - \sin z}{z^3}$, then $z = 0$ is
(a) Essential singularities  (b) Removable singularity  (c) pole  (d) I.E.S

34. The bilinear maps which maps $(0, -i, -1)$ to $(i, 1, 0)$ is
(a) $w = \frac{z+1}{i(z-1)}$  (b) $w = \frac{z+1}{(z-1)}$  (c) $w = \frac{z-1}{(z+1)}$  (d) $w = \frac{i(z+1)}{-z+1}$

35. The series $\frac{1}{3+z}$ is valid for
(a) $\left| \frac{z}{3} \right| < 1$  (b) $\left| \frac{z}{3} \right| < 1$  (c) $\left| \frac{z}{3} \right| > 1$ and $\left| \frac{z}{3} \right| > 1$  (d) $\left| \frac{z}{3} \right| > 1$ and $\left| \frac{z}{3} \right| < 1$

36. The condition for the existance of finite derivative of $f(z)$
(a) continuity is necessary  (b) continuity is sufficient
(c) continuity is necessary and sufficient  (d) continuity is not necessary and sufficient

37. If then Laurentz series expansion of $f(z)$ about $z_0$ can be expressed as $f(z) = \sum_{n=0}^{\infty} a_n (z - z_0)^n$ then $a_n$ is
(a) $\frac{1}{2\pi i} \int_c \frac{f(z)dz}{(z-z_0)^{n+1}}$  (b) $\frac{1}{2\pi i} \int_c \frac{f(z)dz}{(z-z_0)^{n+1}}$  (c) $\int_c \frac{f(z)dz}{(z-z_0)^{n+1}}$  (d) $\int_c \frac{f(z)dz}{(z-z_0)^{n+1}}$

38. The value of the integral $\int_c \frac{dz}{z-3}$ where $c: |z - 1| = 3$ is
(a) $2\pi i$  (b) $-2\pi i$  (c) 0  (d) $\infty$

39. If $f(z)$ and $g(z)$ are analytic inside and on simple closed contour $c$ and if $|g(z)| < |f(z)|$ on $C$. Then $f(z) + g(z)$ and $f(z)$ have same number of zeroes in $C$.this is due to
(a) Rouches theorem  (b) moreras theorem  (c) Cauchy theorem  (d) Lioville's theorem

40. If $f(z)$ is analytic on a rectangle $R$, then $\int_{\partial R} f(z)dz$ is
(a) area of $R$  (b) 0  (c) $\infty$  (d) 1

41. The Geometric series $1 + z + z^2 + z^3 + \ldots$ convergens to
42. \( f(z) = \frac{e^z}{z^2} \) find the nature of the singularities at \( z = 0 \)

(a) Essential singularity  
(b) simple pole at \( z = 0 \)  
(c) double pole at \( z = 0 \)  
(d) Isolated essential

43. The nature of the singularities \( f(z) = \frac{e^{-2z}}{z^3} \) at \( z = 0 \) is

(a) \( z = 0 \) is a pole of order 3 
(b) \( z = 0 \) is a double pole 
(c) \( z = 0 \) is E.S 
(d) \( z = 0 \) is a removable singularity

44. \( f(z) = \frac{\sin z}{z} \), the singularity at \( z = 0 \)

(a) Essential singularity at \( z = 0 \)  
(b) Removable singularity at \( z = 0 \)  
(c) pole at \( z = 0 \)  
(d) None

45. An integral depends only on the end points iff the integral over any closed curve is,

(a) 0  
(b) 1  
(c) \( \infty \)  
(d) \( 2\pi i \)

46. The Cauchy integral formula appears if the winding number is

(a) \( n(\gamma, n) = 0 \)  
(b) \( n(\gamma, a) = 1 \)  
(c) \( n(\gamma, a) = -n(\gamma, a) \)  
(d) \( \infty \)

47. Find the residue of \( f(z) = \frac{1}{z(z-7)^3} \) at \( z = 7 \)

(a) \(-\frac{1}{343}\)  
(b) \(\frac{2}{343}\)  
(c) \(\frac{1}{343}\)  
(d) \(-\frac{2}{343}\)

48. If \( f(z) = \frac{1-e^{2z}}{z^4} \) if \( z = 0 \) is pole of order 3 and its residue is

(a) \(-\frac{4}{3}\)  
(b) \(\frac{4}{3}\)  
(c) \(\frac{2}{3}\)  
(d) \(\frac{2}{3}\)

49. A bilinear transformation having one fixed point called

(a) elliptic transformation  
(b) hyperbolic transformation 
(c) leodromic 
(d) parabolic transformation

50. The radius of convergence of \( \sum_{n=1}^{\infty} \frac{z^n}{n+1} \) is

0000  
(a) 0  
(b) \( \infty \)  
(c) 1  
(d) -1

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