PG-TRB MATHEMATICS

MATERIALS

TIRUVANNAMALAI

UNIT : 9

Complex Analysis

~~STATISTICS - 1~~

CELL : 90925 10255

K. Sellavel.

Complex Analysis - 01

01. If an entire function $f(z)$ is bounded in absolute-value for all z, then $f(z) =$

a) constant b) e^z

c) $1/2$ d) z

02. If $f(z) = \sin z + e^{3z}$, then $\frac{\partial f(z)}{\partial \bar{z}} =$

a) 0 b) $3e^{3\bar{z}}$

c) $\cos \bar{z}$ d) $\cos \bar{z} + 3e^{3\bar{z}}$

03. The residue of $f(z) = \frac{z^2+3z+2}{z^2(z-1)}$ at $z = 0$ is

a) 0 b) -1

c) 2 d) -5

04. If $u = 2x - x^2 + ky^2$ is to be harmonic function then $k =$

a) 0 b) 1

c) 2 d) 3

05. The bilinear transformation which maps $1, i, -1$ to $2, i, -2$ res. is

a) $w = \frac{2i - 6z}{iz - 3}$ b) $w = \frac{2i - 3}{2i - 6}$

c) $w = \frac{2i + 6z}{iz + 3}$ d) $w = \frac{2i + 6z}{iz - 2}$

6. The value of $I = \int_c \frac{z}{z^2-3z+2} dz$ over $|z-2| = \frac{1}{2}$ is

a) $2\pi i$

b) $4\pi i$

c) $6\pi i$

d) $8\pi i$

7. The Taylor series expansion of $f(z) = \frac{z-1}{z+1}$ at $z=1$:

a) $\frac{1}{2}(z-1) - (\frac{1}{2})^2(z-1)^2 + (\frac{1}{2})^3(z-1)^3 - \cdots$

b) $\frac{1}{2}(z-1) + (\frac{1}{2})^2(z-1)^2 + (\frac{1}{2})^3(z-1)^3 + \cdots$

c) $(z-1) - \frac{1}{2}(z-1)^2 + (\frac{1}{2})^3(z-1)^3 + \cdots$

d) $(z-1) + \frac{1}{2}(z-1)^2 + (\frac{1}{2})^2(z-1)^3 + \cdots$

8. The residue of $f(z) = \frac{\sin z}{z\cos z}$ at the pole $z = \pi/2$:

a) 1

b) $-2/\pi$

c) $\pi/2$

d) 0

9. If $f(z)$ has a pole order 3 at $z=a$, then $\text{Res}\{f(z)\} =$

a) $\frac{1}{6} \frac{d^2}{dz^2}\{(z-a)^3 f(z)\}_{z=a}$

b) $\frac{1}{2} \frac{d^2}{dz^2}\{(z-a)^3 f(z)\}_{z=a}$

c) $\frac{1}{6} \frac{d^2}{dz^2}\{(z-a)^2 f(z)\}_{z=a}$

d) $-\frac{1}{2} \frac{d^2}{dz^2}\{(z-a)^2 f(z)\}_{z=a}$

10) If $w = \log z$ then w is not analytic

a) on R b) on -ve real axis

c) at $z=0$ only d) for complex z

11. The principal value of i^i =

a) $e^{-\pi/2}$ b) $e^{\pi/4}$

c) $e^{\pi/2}$ d) $e^{-\pi/4}$

12. The residue of $f(z) = \frac{1}{(z^2+1)^2}$ at $z=i$ equals:

a) $i/4$ b) $-i/4$

c) $i/2$ d) $-i/2$

13. The radius of cgs of power series $\Sigma(1+1/n)^{n^2}$ is

a) e^{-2} b) e^2

c) e d) $1/e$

14. The harmonic conjugate of u is v. Then the harmonic conjugate of v is:

a) u b) $-u$

c) iu d) $-iu$.

15. Given that $f(z) = u+iv$ is analytic and $u = y+e^x \cos y$ then $f(z)$ can be

a) e^z+iz b) e^z-iz

c) ze^{iz} d) e^z+z

16) Suppose α is real and $z = 1-\cos\alpha + i\sin\alpha$ then $|z|$ is

a) $\sqrt{3}\sin(\alpha/2)$ b) $2\sin(\alpha/2)$

c) $\sqrt{2}\cos(\alpha/2)$ d) $2\cos(\alpha/2)$

17 $\int_{|z|=5} \frac{e^{2z}}{(z-1)(z-2)}\, dz =$

a) $2\pi i\,(e^4-e^2)$ b) $2\pi i\,(e^2-e^4)$

c) $2\pi i\, e^2$ d) $2\pi i\, e^4$

18. Let $f(z) = z^3-1$ and C denote the circle of radius 2 with centre as $(1,0)$ oriented anticlockwise. Then the value of $\int_C \frac{f(z)}{z-1}\,dz$ is

a) $2\pi i$ b) 1

c) $4\pi i$ d) 0

19. The roots of the equation $\sin z = \cosh 4$, $z \in \mathbb{C}$

a) $z = n\pi + (-1)^n(\frac{\pi}{2}-4i)$, $n\in \mathbb{Z}$

b) $z = (-1)^n \cdot n\pi + (\frac{\pi}{2}-4i)$, $n \in \mathbb{Z}$

c) $z = n\pi + \pi/2$, $n\in \mathbb{Z}$

d) $z = n\pi + (-1)^n \frac{\pi}{2}$, $n \in \mathbb{Z}$

20. Consider the complex valued function $f(z) = (z-1)^2\, e^{\frac{1}{(z-1)^2}}$. Then $f(z)$ has

a) R.S at 1 b) pole at 0 of order 2

c) pole at 1 of order 2 d) E.S at 1.

21. If S is the +ve oriented circle $|z-3i|=2$

$\int_S \frac{dz}{z^2+4}$?

a) $-\pi/2$

b) $\pi/2$

c) $-i\pi/2$

d) $i\pi/2$

22. At $z=0$, $\frac{e^z}{z(1-e^{-z})}$ has which one of the following?

(a)

a) R.S

b) pole-order 1

c) pole of order 2

d) E.S.

23. For what value of z does $\sum_{n=0}^{\infty} \frac{1}{3^n}(z-1)^{2n}$ cgs

a) $|z| \le 3$

b) $|z| \le \sqrt{3}$

c) $|z-1| < \sqrt{3}$

d) $|z-1| \le \sqrt{3}$

24. The bilinear transformation which maps the pts $z=0,1,-1$ into $w=i,\infty,0$ is .

a) $i\left(\frac{z+1}{z-1}\right)$

b) $\frac{z-1}{z+1}$

c) $\frac{z+1}{z-1}$

d) $-i\left(\frac{z+1}{z-1}\right)$

25. Let $f(z) = \frac{1+2z}{z^2+z^3}$ which one of the following is true?

a) $f(z) = \frac{1}{z^2}+\frac{1}{z^2}+1+z-z^2+z^3+\dots$ for $0<|z|<1$

b) Residue of $f(z)$ at $z=0$ is 1

c) $f(z)$ is analytic at $z=0$

d) Residue of $f(z)$ at $z=0$ is -1

1 - a	6 - b	11 - a	16	21 -
2 a	7 - a	12 - b	17 - a	22 - c
3 - d	8 - b	13 - c	18 - d	23 - d
4 - c	9 - b	14 - b	19	24 - d
5 - a	10 - c	15 - b	20 - d	25 - a, b

soln

7) $f(z) = \frac{z-1}{z+1}$

$= \frac{z-1}{2+(z-1)} = \frac{z-1}{2}\left(\frac{1}{1+\left(\frac{z-1}{2}\right)}\right)$

$= \frac{z-1}{2}\left(1-\left(\frac{z-1}{2}\right)+\left(\frac{z-1}{2}\right)^2 - \cdots\right)$

Ans: a

8) $\text{Res } f(z) = \text{Res} \frac{\sin z}{2\cos z}$

$= \underset{z=\pi/2}{\text{Res}} \frac{\sin z}{2(1-\sin z)}$ (verify)

$= \left(\frac{1}{2}\right)\pi/2$

Ans: b

$= \frac{-2}{\pi}$

19. $\frac{d}{dz} \log z = \frac{1}{z}$

= it is not analytic only when $z=0$.

11)

$z = 1-\cos\alpha + i\sin\alpha$

$|z|^2 = (1-\cos\alpha)^2 + \sin^2\alpha$

$= 1 - 2\cos\alpha + \cos^2\alpha + \sin^2\alpha$

$= 2 - 2\cos\alpha$

$= 2(1-\cos\alpha)$

$= 2\left(2\sin^2\frac{\alpha}{2}\right)$

15) u given ∴ put $x = \frac{z}{2}, y = -\frac{iz}{2}$

$$f(z) = -\frac{iz}{2} + e^{\frac{z}{2}} \cos\left(-\frac{iz}{2}\right)$$

$$f(z) = \int (u_x(z,0) - i u_y(z,0))\, dz$$

$$= \int (e^{z} \cos 0 - i(1 + e^{z}(-\sin 0))\, dz$$

$$= e^{z} - iz$$

25)
$$f(z) = \frac{1+2z}{z^2+z^3}$$

$$= \frac{1+2z}{z^2}\left(\frac{1}{1+z}\right)$$

$$= \frac{1+2z}{z^2}\left(1 - z + z^2 - z^3 + z^4 - \dots\right)$$

$$= \left(\frac{1}{z^2} + \frac{2}{z}\right)\left(1 - z + z^2 - z^3 + z^4\right)$$

$$= \frac{1}{z^2} - \cancel{\frac{1}{z}} + \frac{2}{z} - 2 + 1 \cancel{-z} - 2z^2$$

$$= \frac{1}{z^2} + \frac{1}{z} - 1$$

$$= \left(\frac{1}{z^2} - \frac{z}{z} + 1 - z + z^2 + \right) + \left(\frac{2}{z} - 2 + 2z - 2z^2\right.$$

$$= \frac{1}{z^2} + \frac{1}{z} - 1 + z - z^2 + \dots$$

$$f(z) = \frac{1+2z}{z^2(1+z)}$$

$$= \frac{d}{dz}\left(\frac{1+2z}{1+z}\right)_{at\ z=0}$$

$$= \left(\frac{(1+z)(2) - (1+2z)(1)}{(1+z)^2}\right)_0$$

$$= \frac{2-1}{1}$$

$$= 1$$