

NATIONAL ACADEMY

DHARMAPURI

TRB MATHEMATICS

DIFFERENTIAL EQUATIONS

‘Material Available with Question papers’

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TEST BACTH –SATURDAY & SUNDAY

**NATIONAL ACADEMY
DHARMAPURI**

Unit-VIII - Differential Equations

Linear differential equation - constant co-efficients - Existence of solutions - Wronskian - independence of solutions - Initial value problems for second order equations - Integration in series - Bessel's equation - Legendre and Hermite Polynomials - elementary properties - Total differential equations - first order partial differential equation - Charpits method

DIFFERENTIAL EQUATIONS

Definition:

An equation involving one dependent variable and its derivatives with respect to one or more independent variables is called a **Differential Equation**.

Differential equation are of two types.

- (i) Ordinary Differential equation
- (ii) Partial Differential equation

Ordinary Differential equation

An ordinary differential equation is a differential equation in which a single independent variable enters either explicitly or implicitly.

$$(i) \frac{dy}{dx} = x + 5 \quad (ii) (y')^2 + (y'')^3 + 3y = x^2 \quad (iii) \frac{d^2y}{dx^2} - 4 \frac{dy}{dx} + 3y = 0$$

Partial Differential equation

A partial differential equation is one in which at least two independent variable occur.

Example :

$$x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = z$$

Order

The **order** of a differential equation is the order of the highest differential coefficient **degree**

The **degree** of the differential equation is the degree of the highest order derivative after removing radicals and fractions

Problems

Find the order and degree of the differential equation

$$1. \left[1 + \left(\frac{dy}{dx} \right)^2 \right]^3 = \left(\frac{d^2y}{dx^2} \right)^2$$

order = 2 , degree = 2

$$2. \cos x \frac{d^2y}{dx^2} + \sin x \left(\frac{dy}{dx} \right)^2 + 8y = \tan x$$

order = 2 , degree = 1

$$3. \frac{d^2y}{dx^2} + \sqrt{1 + \left(\frac{dy}{dx}\right)^3} = 0$$

order = 2 , degree = 2

$$4. \frac{d^2y}{dx^2} = [4 + \left(\frac{dy}{dx}\right)^2]^{\frac{3}{4}}$$

order = 2 , degree = 4

$$5. \frac{d^2y}{dx^2} + x = \sqrt{y + \left(\frac{dy}{dx}\right)}$$

order = 1 , degree = 2

$$6. (1 + y')^2 = y'^2$$

order = 1 , degree = 1

$$7. y = 4\frac{dy}{dx} + 3x\frac{dx}{dy}$$

order = 1 , degree = 2

$$8. \sin x(dx+dy) = \cos x(dx-dy)$$

order = 1 , degree = 1

$$9. \frac{d^3y}{dx^3} + \left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^5 + y = 7$$

order = 3 , degree = 1

$$10. \frac{d^4x}{dt^4} + \frac{d^2x}{dt^2} + \left(\frac{dx}{dt}\right)^5 = e^t \quad (\text{TRB})$$

order = 4 , degree = 1

Formation of differential equations :

Let $f(x, y, c_1) = 0$ be an equation containing x, y and one arbitrary constant c_1 . If c_1 is eliminated by differentiating $f(x, y, c_1) = 0$ with respect to the independent variable once, we get a relation involving x, y and $\frac{dy}{dx}$

If we have an equation $f(x, y, c_1, c_2) = 0$ containing two arbitrary constants c_1 and c_2 , then by differentiating this twice, we get three equations (including f). If the two arbitrary constants c_1 and c_2 are eliminated from these equations, we get a differential equation of second order.

Problems

Form the differential equation by eliminating arbitrary constant

$$1. y = ax + a^2$$

Differentiating with respect 'x'

$$\frac{dy}{dx} = a$$

differential equation is, $y = \frac{dy}{dx} x + \left(\frac{dx}{dt}\right)^2$

2. $y = A\cos x + B\sin x$

Differentiating with respect 'x'

$$\frac{dy}{dx} = -A\sin x + B\cos x$$

Again Differentiating with respect 'x'

$$\frac{d^2y}{dx^2} = -A\cos x - B\sin x = -(A\cos x + B\sin x) = -y$$

$$\frac{d^2y}{dx^2} + y = 0$$

3. $y = A\cos kx + B\sin kx$

Differentiating with respect 'x'

$$\frac{dy}{dx} = -Ak\sin x + Bk\cos x$$

Again Differentiating with respect 'x'

$$\frac{d^2y}{dx^2} = -Ak^2\cos x - Bk^2\sin x = -k^2(A\cos x + B\sin x) = -k^2y$$

$$\frac{d^2y}{dx^2} + k^2y = 0$$

4. $y = A\cos 4x + B\sin 4x$

Differentiating with respect 'x'

$$\frac{dy}{dx} = -4A\sin x + 4B\cos x$$

Again Differentiating with respect 'x'

$$\frac{d^2y}{dx^2} = -16A\cos x - 16B\sin x = -16(A\cos x + B\sin x) = -16y$$

$$\frac{d^2y}{dx^2} + 16y = 0$$

5. $y^2 = Ax^2 + Bx + C$

Differentiating with respect 'x'

$$2yy' = 2Ax + B$$

Again Differentiating with respect 'x'

$$2[y'y'' + y'y'] = 2A$$

Again Differentiating with respect 'x'

$$yy''''+3y''y'+2y'y'' = 0 \Rightarrow yy''''+3y''y' = 0 \text{ (or) } y \frac{d^3y}{dx^3} + 3 \frac{d^2y}{dx^2} \times \frac{dy}{dx} = 0$$

$$6. y = Ae^{3x} + Be^{-3x}$$

$$y' = 3Ae^{3x} - 3Be^{-3x}$$

$$y'' = 9Ae^{3x} + 9Be^{-3x} = 9(Ae^{3x} + Be^{-3x})$$

$$y'' = 9y \Rightarrow y'' - 9y = 0$$

$$7. y = Ae^{3x} + Be^{5x} \text{ ----- (1)}$$

$$y' = 3Ae^{3x} + 5Be^{5x} \text{ ----- (2)}$$

$$y'' = 9Ae^{3x} + 25Be^{5x} \text{ ----- (3)}$$

From equation (1),(2) and (3) eliminating A,B

$$\begin{vmatrix} y & 1 & 1 \\ y' & 3 & 5 \\ y'' & 9 & 25 \end{vmatrix} = 0$$

$$y'' - 8y' + 15 = 0 \text{ (OR)}$$

$$y = Ae^{p_1x} + Be^{p_2x} \text{ is a solution of } (D-p_1)(D-p_2)y = 0$$

$$(D-3)(D-5)y = 0 \Rightarrow (D^2-8D+15)y = 0 \text{ (or) } \frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 15y = 0$$

$$8. y = c_1e^{-x} + c_2e^{2x} \text{ (TRB)}$$

$$(D-1)(D+2)y = 0$$

$$(D^2-D+2)y=0 \Rightarrow \frac{d^2y}{dx^2} - \frac{dy}{dx} + 2y = 0$$

$$9. y = Ae^{2x} + Be^{-2x}$$

$$(D+2)(D-2)y = 0$$

$$(D^2+4)y=0 \Rightarrow \frac{d^2y}{dx^2} + 4y = 0$$

$$10. y = e^{3x}(C\cos 2x + D\sin 2x)$$

$$ye^{-3x} = C\cos 2x + D\sin 2x \text{ ----- (1)}$$

Differentiating with respect 'x'

$$y(-3e^{-3x}) + e^{-3x}y' = -2C\sin 2x + 2D\cos 2x$$

$$e^{-3x}[-3y + y'] = -2C\sin 2x + 2D\cos 2x$$

Again Differentiating with respect 'x'

$$e^{-3x}(-3y' + y'') - 3(3y + y')e^{-3x} = -4C\cos 2x - 4D\sin 2x = -4(C\cos 2x + D\sin 2x)$$

$$e^{-3x}[y'' - 6y' + 9y] = -4ye^{-3x} \Rightarrow y'' - 6y' + 13y = 0 \text{ (OR)}$$

If $p = \alpha + i\beta$, then Differential equation,

$$(D^2 - (\text{sum of roots})D + \text{Product of roots})y = 0$$

$$(D^2 - (3+i2+3-i2)D + (3+i2)(3-i2))y = 0$$

$$(D^2 - 6D + 13)y = 0$$

$$11. y = (Ax+B)e^{3x}$$

$$\text{Differential equation is, } (D-3)(D-3)y = 0$$

$$(D^2 - 9)y = 0$$

Solutions of first order and first degree equations:

- (i) Variable separable method
- (ii) Homogeneous differential equation
- (iii) Linear differential equation

Variable separable :

Variables of a differential equation are to be rearranged in the form

$$f_1(x) g_2(y) dx + f_2(x) g_1(y) dy = 0$$

i.e., the equation can be written as

$$f_2(x)g_1(y)dy = - f_1(x) g_2(y) dx$$

$$\frac{g_1(y)}{g_2(y)} dy = \frac{f_1(x)}{f_2(x)} dx \text{ and integrating on both sides}$$

$$\int \frac{g_1(y)}{g_2(y)} dy = \int \frac{f_1(x)}{f_2(x)} dx$$

$$1. \frac{dy}{dx} + \left(\frac{1-y^2}{1-x^2}\right)^{\frac{1}{2}} = 0$$

$$\frac{dy}{\sqrt{1-y^2}} = \frac{dx}{\sqrt{1-x^2}}$$

$$\text{integrating } \int \frac{dy}{\sqrt{1-y^2}} = \int \frac{dx}{\sqrt{1-x^2}}$$

$$\sin^{-1}y - \sin^{-1}x = c$$

$$2. \text{ Solve } \frac{dy}{dx} = y^2 x^3$$

$$\frac{dy}{y^2} = \frac{dx}{x^3}$$

$$\text{integrating } \int \frac{dy}{y^2} = \int x^3 dx$$

$$\frac{1}{y} = \frac{x^4}{4} + c$$

$$3. \text{ Solve } \frac{dy}{dx} = \frac{y+2}{x-1}$$

$$\frac{dy}{y+2} = \frac{dx}{x-1}$$

$$\text{integrating } \int \frac{dy}{y+2} = \int \frac{dx}{x-1}$$

$$\log(y+2) = \log(x-1) + \log c$$

$$\log\left(\frac{y+2}{x-1}\right) = \log c$$

$$\frac{y+2}{x-1} = c$$

4. Solve $e^x \tan y dx + (1-e^x) \sec^2 y dy = 0$

$$\frac{e^x}{1-e^x} dx = -\frac{\sec^2 y dy}{\tan y}$$

$$\text{integrating } \int \frac{e^x}{1-e^x} dx = -\int \frac{\sec^2 y dy}{\tan y}$$

$$-\log(1-e^x) = -\log \tan y + \log c$$

$$\log\left(\frac{\tan y}{1-e^x}\right) = \log c$$

$$\frac{\tan y}{1-e^x} = \log c$$

5. Solve $x\sqrt{1+y^2} + y\sqrt{1+x^2} \frac{dy}{dx}$

$$\frac{x dx}{\sqrt{1+x^2}} = -\frac{y dy}{\sqrt{1+y^2}}$$

$$\text{integrating } \int \frac{x dx}{\sqrt{1+x^2}} = -\int \frac{y dy}{\sqrt{1+y^2}}$$

$$\times 2 \Rightarrow \int \frac{2x dx}{\sqrt{1+x^2}} = -\int \frac{2y dy}{\sqrt{1+y^2}}$$

$$2\sqrt{1+x^2} = -2\sqrt{1+y^2} + c$$

6. Solve $\sqrt{1-x^2} \sin^{-1} x dy + y dx = 0$

$$\frac{dy}{y} = -\frac{dx}{\sin^{-1} x \sqrt{1-x^2}}$$

$$\text{integrating } \int \frac{dy}{y} = -\int \frac{dx}{\sin^{-1} x \sqrt{1-x^2}}$$

$$\int \frac{dy}{y} = -\int \frac{du}{u} ; \text{ let } u = \sin^{-1} x, du = \frac{1}{\sqrt{1-x^2}} dx$$

$$\log y = -\log u + \log c$$

$$y u = c \Rightarrow y \sin^{-1} x = c$$

7. Solve $y dx - x dy + 3x^2 y^2 e^{x^3} dx = 0$

$$\frac{y dx - x dy}{y^2} + 3x^2 e^{x^3} dx = 0$$

$$d\left(\frac{x}{y}\right) + 3x^2 e^{x^3} dx = 0$$

$$\text{integrating } \int d\left(\frac{x}{y}\right) + \int 3x^2 e^{x^3} dx = 0$$

$$\frac{x}{y} + e^{x^3} = c$$

Homogeneous Differential Equations

A differential equation of the form $\frac{dy}{dx} = \frac{f(x,y)}{g(x,y)}$ OR $\frac{dy}{dx} = f\left(\frac{y}{x}\right)$ is called a Homogeneous equation. Where $f(x,y)$ and $g(x,y)$ are homogeneous equation of the same degree.

$$\text{Ex: } \frac{dy}{dx} = \frac{2xy}{x^2+y^2}$$

$$\text{In such case, } y = vx \Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$$

$$1. \text{ Solve } \frac{dy}{dx} = \frac{y}{x} + \tan \frac{y}{x}$$

$$\text{Put } y = vx \Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$$

$$v + x \frac{dv}{dx} = v + \tan v$$

$$x \frac{dv}{dx} = \tan v$$

$$\frac{dv}{\tan v} = \frac{dx}{x}$$

$$\text{integrating } \int \frac{dv}{\tan v} = \int \frac{dx}{x}$$

$$\log \sin v = \log x + \log c$$

$$\sin v = xc, \text{ put } v = \frac{y}{x}$$

$$\sin \frac{y}{x} = xc$$

$$2. \text{ Solve } \frac{dy}{dx} + \frac{y}{x} = \frac{y^2}{x^2}$$

$$\text{Put } y = vx \Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$$

$$v + x \frac{dv}{dx} + v = v^2$$

$$x \frac{dv}{dx} = v^2 - 2v$$

$$\frac{dv}{v^2 - 2v} = \frac{dx}{x}$$

$$\frac{dv}{v(v-2)} = \frac{dx}{x} \quad \left\{ \frac{1}{v(v-2)} = \frac{A}{v} + \frac{B}{v-2} \right\}, A = -\frac{1}{2}, B = \frac{1}{2}$$

$$-\frac{1}{2} \int \frac{dv}{v} + \frac{1}{2} \int \frac{dv}{v-2} = \int \frac{dx}{x}$$

$$-\frac{1}{2} \log v + \frac{1}{2} \log(v-2) = \log x + \log c$$

$$\frac{v-2}{v} = x^2 c, \text{ put } v = \frac{y}{x}$$

$$v-2 = vx^2 c$$

$$\frac{y}{x} - 2 = \frac{y}{x} \times x^2 c$$

$$y - 2x = yx^2 c$$

3. On putting $y = vx$, the homogenous differential equation $x^2 dy + y(x + y) dx = 0$

$$\text{Becomes } xdv + (2v + v^2) dx = 0$$

4. Solve $x dy - y dx = \sqrt{x^2 + y^2} dx$

$$\text{Ans: } xdv - \sqrt{1 + v^2} dx = 0$$

5. Solve $\frac{dy}{dx} = \frac{y^3 + 3x^2 y}{x^3 + 3x y^2}$

$$\text{Ans: } \frac{dx}{x} = \frac{(1+3v^2)}{2v(v^2-1)} dv$$

Linear differential equations

➤ The linear equation in y of the first order is of the form $\frac{dy}{dx} + Px = Q(x)$, where P, Q are functions in x

$$\text{The general solution is } y(I.F) = \int Q(I.F) dx + c$$

$$\text{Where Integral Factor(I.F)} = e^{\int P dx}$$

➤ The linear equation in x of the first order is of the form $\frac{dx}{dy} + Py = Q(y)$, where P, Q are functions in y

$$\text{The general solution is } yx(I.F) = \int Q(I.F) dy + c$$

$$\text{Where Integral Factor(I.F)} = e^{\int P dy}$$

Note :

Integral factor of D.E is (I.F)

$$\text{Then, } P = \frac{(I.F)'}{I.F}$$

1. Find the integral factor of $(1-x^2) \frac{dy}{dx} + 2xy = x\sqrt{1-x^2}$

$$\div (1-x^2) \Rightarrow \frac{dy}{dx} + \frac{2x}{(1-x^2)} y = \frac{x\sqrt{1-x^2}}{(1-x^2)}$$

$$P = \frac{2x}{(1-x^2)}, Q = \frac{x\sqrt{1-x^2}}{(1-x^2)}$$

$$I.F = e^{\int P dx} = e^{\int \frac{2x}{(1-x^2)} dx} = e^{-\log(1-x^2)} = \frac{1}{(1-x^2)}$$

2. Find I.F of $\frac{dy}{dx} + y \cot x = \sin 2x$

$$I.F = e^{\int P dx} = e^{\int \cot x dx} = e^{\log \sin x} = \sin x$$

3. Find I.F of $(1+x^2) \frac{dy}{dx} + y = e^{\tan^{-1} x}$

$$\div (1+x^2) \Rightarrow \frac{dy}{dx} + \frac{1}{(1+x^2)}y = \frac{e^{\tan^{-1}x}}{(1+x^2)}$$

$$I.F = e^{\int \frac{1}{(1+x^2)}dx} = e^{\tan^{-1}x}$$

4. Find the solution of $\frac{dy}{dx} + \frac{2x}{(1+x^2)}y = 0$

$$I.F = e^{\int \frac{2x}{(1+x^2)}dx} = e^{\log(1+x^2)} = (1+x^2)$$

The solution is, $y(I.F) = \int Q(I.F)dx + c$

$$y(1+x^2) = \int 0dx + c$$

$$y(1+x^2) = c$$

5. If the integral factor of D.E is $\sec x$, find $P(x)$

$$P(x) = \frac{(I.F)'}{I.F} = \frac{\sec x \tan x}{\sec x} = \tan x$$

6. If e^x is an integral factor of D.E. Find $P(x)$

$$P(x) = \frac{(I.F)'}{I.F} = \frac{e^x}{e^x} = 1$$

7. If $e^{f(x)}$ is IF of D.E, then $P(x) = f'(x)$

Exercise

1. Solve $\frac{dy}{dx} + y = x$, Ans: $e^x(y - x + 1) = c$

2. Find IF of $\frac{dy}{dx} + xy = x$, ans: $e^{\frac{x^2}{2}}$

3. Find IF of $\frac{dy}{dx} + \frac{y}{x} = \sin(x^2)$ Ans: x

4. Solve $\frac{dy}{dx} + 2y \tan x = \sin x$, Ans: $y = \cos x + C \cos^2 x$

5. Find I.F of $\frac{dy}{dx} - 3y = 6$, Ans: e^{-3x^2}

6. Find the solution of $\frac{dy}{dx} + 2y - 3 = 0$, Ans: $ye^{2x} = \frac{3e^{2x}}{2} + c$

Bernoulli's equation

An equation of the form $\frac{dy}{dx} + py = Qy^n$, Where P, Q are functions in x only.

$$\div y^n \Rightarrow y^{-n} \frac{dy}{dx} + Py^{1-n} = Q \text{ -----(1)}$$

Put $z = y^{1-n}$

$$\frac{dz}{dx} = (1-n)y^{-n} \frac{dy}{dx}$$

$$\frac{1}{(1-n)} \frac{dz}{dx} = y^{-n} \frac{dy}{dx}$$

$$(1) \Rightarrow \frac{1}{(1-n)} \frac{dz}{dx} + Pz = Q, \text{ which is linear equation in } z$$

$$1. \text{ Solve, } (x+1) \frac{dy}{dx} + 1 = 2e^{-y}$$

$$\div e^{-y} \Rightarrow e^y (x+1) \frac{dz}{dx} + e^y = 2 \text{ ----- (1)}$$

$$\text{Let } z = e^y \Rightarrow \frac{dz}{dx} = e^y \frac{dy}{dx}$$

$$(1) \Rightarrow (x+1) \frac{dz}{dx} + z = 2$$

$$\div (x+1) \Rightarrow \frac{dz}{dx} + \frac{1}{(x+1)} z = \frac{2}{(x+1)}$$

$$I.F = e^{\int \frac{dx}{(x+1)}} = (x+1)$$

The general solution is, $ze^{\int P dx} = \int Qe^{\int P dx} dx + c$

$$Z(x+1) = \int \frac{2}{(x+1)} (x+1) dx + c$$

$$Z(x+1) = 2x + c$$

$$2. \text{ Find the I.F of } \frac{dy}{dx} (x^2 y^3 + xy) = 1$$

$$x^2 y^3 + xy = \frac{dx}{dy}$$

$$\div x^2 \Rightarrow \frac{1}{x^2} \frac{dx}{dy} \frac{y}{x} = y^3 \text{ ----- (1)}$$

$$\text{put } z = \frac{1}{x}$$

$$\frac{dz}{dy} = \frac{1}{x^2} \frac{dx}{dy}$$

$$(1) \Rightarrow \frac{dz}{dy} + zy = y^3$$

$$(2) I.F = e^{\int P dy} = e^{\int y dy} = e^{\frac{y^2}{2}}$$

Exercise

$$1. \text{ Find I F of } \frac{dy}{dx} + xtany = x^3, \text{ Ans: } e^{\frac{x^2}{2}}$$

$$2. \text{ Find the I.F of } \frac{dy}{dx} + y \cos x = y^n \sin 2x, \text{ Ans: I F} = e^{-(1-n)\sin x}$$

Equation of the first order with higher degree

We shall denote $\frac{dy}{dx}$ by 'p'

The D.E of the first order and nth degree, $p^n + A_1 p^{n-1} + A_2 p^{n-2} + \dots + A_n = 0$

Where $A_1, A_2, A_3, \dots, A_n$ denote functions of x and y

In this case resolve the given equation into factor of first degree,

$$(p-R_1)(p-R_2)(p-R_3) \dots (p-R_n) = 0$$

This implies, $p=R_1, p=R_2, p=R_3, \dots, p=R_n$

Solve the equation, then we get,

$$\varphi_1(x, y, c_1) = 0, \varphi_2(x, y, c_2) = 0, \varphi_3(x, y, c_3) = 0, \dots, \varphi_n(x, y, c_n) = 0,$$

Where $c_1, c_2, c_3, \dots, c_n$ are constant

The solution of D.E is, $\varphi_1(x, y, c_1) \varphi_2(x, y, c_2) \varphi_3(x, y, c_3) \dots \varphi_n(x, y, c_n) = 0$

1. Solve, $p^2 - 5p + 6 = 0$ (Or) $\left(\frac{dy}{dx}\right)^2 - 5\frac{dy}{dx} + 6 = 0$

$$\Rightarrow (p-2)(p-3) = 0$$

$$p = 2, p = 3$$

$$\frac{dy}{dx} = 2, \frac{dy}{dx} = 3$$

$$dy = 2dx, \quad dy = 3dx$$

$$\text{Integrating, } y = 2x + c_1, y = 3x + c_2$$

$$y - 2x - c_1 = 0, y - 3x - c_2 = 0$$

$$\text{The solution is, } (y - 2x - c_1)(y - 3x - c_2) = 0$$

2. Solve, $p(p-y) = x(x+y)$

$$p^2 - py - x^2 - xy = 0$$

$$p^2 - x^2 - y(x+p) = 0$$

$$(p+x)(p-x) - y(x+p) = 0$$

$$(p+x)(p-x-y) = 0$$

$$P = -x, p = x+y$$

$$\frac{dy}{dx} = -x \Rightarrow y = -\frac{x^2}{2} + c_1 \Rightarrow (2y + x^2 - c_1) = 0$$

$$\frac{dy}{dx} = x+y \Rightarrow \frac{dy}{dx} - y = x$$

$$I.F = e^{\int -dx} = e^{-x}$$

$$\text{The solution, } ye^{-x} = \int x e^{-x} dx + c_2$$

$$ye^{-x} = -xe^{-x} - e^{-x} + c_2 \Rightarrow (y+x+1 - c_2e^{-x}) = 0$$

$$\text{the solution is, } \Rightarrow (2y+x^2 - c_1)(y+x+1 - c_2e^{-x}) = 0$$

3. Solve , $x^2p^2 + 3xyp + 2y^2 = 0$ (TRB)

$$x^2p^2 + 2xyp + 3xyp + 2y^2 = 0$$

$$xp(xp + 2y) + y(xp + 2y) = 0$$

$$(px + y)(xp + 2y) = 0$$

$$Px = -y, xp = -2y$$

$$\frac{dy}{dx} x = -y \Rightarrow \log y = -\log x + \log c_1 \Rightarrow (xy - c_1) = 0$$

$$xp = -2y \Rightarrow \frac{dy}{dx} x = -2y \Rightarrow \log y = -2\log x + \log c_1 \Rightarrow (yx^2 - c_2) = 0$$

$$\text{The solution is, } (xy - c_1)(yx^2 - c_2) = 0$$

Exercise

4. Solve , $p^2 - 7p + 10 = 0$,Ans: $(y - 2x - c_1)(y - 5x - c_2) = 0$

5. Solve, $p^2 - 7p + 12 = 0$,Ans: $(y - 4x - c_1)(y - 3x - c_2) = 0$

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DIFFERENTIAL EQUATIONS TEST -1

1. What is the order and degree of the differential equation $\frac{d^2y}{dx^2} + \sqrt{1 + \left(\frac{dy}{dx}\right)^2} = 0$

(a) First order, second order

(b) second order, first degree

(c) first order, first degree

(d) second order, second degree

2. The differential equation derive from $y = Ae^{2x} + Be^{-2x}$ have the order, when A, B are constants

(a) 3

(b) 2

(c) 1

(d) None of these

3. The differential equation of $y = Ae^{3x} + Be^{5x}$ is

(a) $y'' - 8y + 15y = 0$

(b) $y'' + 8y + 15y = 0$

(c) $y'' + 8y = 0$

(d) $y'' = 0$

4. The differential equation of $x = A \cos(pt - \alpha)$ is

(a) $\frac{d^2x}{dt^2} = 0$

(b) $\frac{d^2x}{dt^2} = -p^2x$

(c) $\frac{d^2x}{dt^2} = 0$

(d) $\frac{dx}{dt} = -px$

5. The solution of $\frac{dy}{dx} = 2xy$ is

(a) $c = \log y + x$

(b) $c = \log y - x^2$

(c) $c = xy$

(d) $c = y \log x$

6. The degree of the differential equation of $\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 + \sin x + 1 = 0$ is

(a) 1

(b) 2

(c) 3

(d) 4

7. The number of arbitrary constants in the general solution of differential equation of fourth order is

(a) 1

(b) 2

(c) 3

(d) 4

8. A homogeneous differential equation of the form $\frac{dx}{dy} = f\left(\frac{x}{y}\right)$ can be solve by making the substitution
 (a) $y = vx$ (b) $v = yx$ (c) $x = vy$ (d) $x = v$
9. Order and degree of differential equation are always
 (a) positive integer (b) Negative integer (c) integer (d) None of these
10. The differential equation of $y = A\cos x - B\sin x$ is
 (a) $y'' - y = 0$ (b) $y'' + y = 0$ (c) $y' + y = 0$ (d) $y'' + x y = 0$
11. The differential equation of $y = Ae^x + Be^{-x} + 3x$ is
 (a) $y'' + x y = 0$ (b) $y'' + y = 3x$ (c) $y'' - y = -3x$ (d) $y'' + y = 2x$
12. Order and degree of differential equation $\left(\frac{d^2y}{dx^2}\right)^{\frac{1}{2}} = \left(x - \frac{dy}{dx}\right)^{\frac{1}{3}}$
 (a) (3,2) (b) (1,3) (c) (2,1) (d) (2,3)
13. The solution of the differential equation $\frac{dy}{dx} = \frac{y+1}{x-1}$ is
 (a) $c = \frac{y+1}{x-1}$ (b) $c = \frac{y+1}{x+1}$ (c) $c = (y+1)(x-1)$ (d) $c(x-1) = y$
14. The general solution of differential equation $\frac{dy}{dx} = y^2 x^3$
 (a) $y = x^3 + k$ (b) $y = x^4 + k$ (c) $y = \frac{-4}{x^4 + k}$ (d) $y = x^3 + e^{2x} + k$
15. The solution of the differential equation $\frac{dy}{dx} = e^{2x-y} + x^3 e^{-y}$ is
 (a) $x(2x-y) + y(3-y) = c$ (b) $4e^y = 2e^{2x} + x^4 + c$
 (c) $2e^{2x} + e^y = c$ (d) $2e^{2x} + e^y + x^4 = c$
16. The general solution of differential equation $\sin x \cos y dx - \cos x \sin y dy = 0$
 (a) $\sin x = x \sec y$ (b) $x \sec x = c \sec y$ (c) $\sec x = c \sec y$ (d) $\sec x = c \sec y + c$
17. The particular solution of $y' = xe^x$ with initial condition $y = 3$ when $x = 1$
 (a) $x = ye^x - e^x + 3$ (b) $y = xe^x - e^x + 1$ (c) $y = xe^x - e^x + 3$ (d) $y = e^x(x+1) + 3$
18. A curve passes through the point (0,0) with differential equation $y' = e^{3x-2y}$, then the equation of the curve is
 (a) $3e^x = ye^{3x} + 1$ (b) $2e^{2y} = 3e^{3x} + 1$ (c) $3e^{2y} = 2e^{3x}$ (d) $3e^{2y} = 2e^{3x} + 2$
19. The general solution of differential equation $\frac{dy}{dx} = e^x$
 (a) $y = e^x + c$ (b) $y = e^{-x} + c$ (c) $y = -e^x + c$ (d) $x = \log y + c$
20. The differential equation of the family parabolas $y^2 = 4ax$ is
 (a) $\frac{dy}{dx} = 4\left(\frac{dy}{dx}\right)^2$ (b) $y = 2x \frac{dy}{dx}$ (c) $\frac{d^2y}{dx^2} = 4$ (d) none of these
21. The differential equation of the family of straight lines is
 (a) $\frac{dy}{dx} = 4\left(\frac{dy}{dx}\right)^2$ (b) $y = 2x \frac{dy}{dx}$ (c) $\frac{d^2y}{dx^2} = 0$ (d) None of these
22. Differential equation of the family of circles with center at origin & radius a is
 (a) $X - y \frac{dy}{dx} = 0$ (b) $y - x \frac{dy}{dx} = 0$ (c) $x + y \frac{dy}{dx} = 0$ (d) $y + x \frac{dy}{dx} = 0$

23. Differential equation of the family of circles which passes through the origin & whose centers are on the x-axis is

(a) $2xy \frac{dy}{dx} + x^2 + y^2 = 0$ (b) $2xy \frac{dy}{dx} + x^2 - y^2 = 0$ (c) $x \frac{dy}{dx} + x^2 - y^2 = 0$ (d) $2y \frac{dy}{dx} + x^2 - y^2 = 0$

24. The solution of $(1 + x^2)dy = (1 + y^2)dx$ is

(a) $\tan^{-1}y - \tan^{-1}x = \tan^{-1}c$

(b) $\tan^{-1}y + \tan^{-1}x = \tan^{-1}c$

(c) $\tan^{-1}x - \tan^{-1}y = \tan^{-1}c$

(d) $\tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}c$

25. Solution of $(xy^2 + x)dx + (xy^2 + y)dy = 0$ is

(a) $(x^2 + 1) = c(y^2 + 1)$

(b) $(x^2 - 1) = c(y^2 - 1)$

(c) $(x^2 + 1)(y^2 + 1) = c$

(d) $(x^2 - 1)(y^2 - 1) = c$

-Material Available with Question papers-

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