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Unit-II - Real Analysis

Cardinal numbers - Countable and uncountable cordinals - Cantor's diagonal process – Properties of real numbers - Order - Completeness of R-Lub property in R-Cauchy sequence - Maximum and minimum limits of sequences - Topology of R.Heine Borel - Bolzano Weierstrass - Compact if and only if closed and bounded - Connected subset of R-Lindelof's covering theorem - Continuous functions in relation to compact subsets and connected subsets- Uniformly continuous function – Derivatives - Leftand right derivatives - Mean value theorem - Rolle's theorem- Taylor's theorem- L' Hospital's Rule -Riemann integral - Fundamental theorem of Calculus –Lebesgue measure and Lebesque integral onR'Lchesque integral of Bounded Measurable function - other sets of finite measure - Comparison of Riemann and Lebesque integrals - Monotone convergence theorem - Repeated integrals.

CLASS -I

Cardinal Numbers

The total number of element in a set is called the cardinal number.

The cardinal number of a set A is denoted by n(A)

1. If A ={1,2,3,……,n},then the cardinal number of A is n(A) = n
2. If n(A) =n(B),then A and B arecalled Equivalent sets.
3. The ardinal number of power set of A with n element is n[p(A)] = 2^n
4. n(A$\cup B$) = n(A)+n(B)-n(A$\cap B$)

One to one function

The function f: A$\longrightarrow$ B is called one to one function if f(x) = f(y) ⇒ x = y for all x,y∈ A.

Examble :

1. f(x) = e^x (-$\infty < x < \infty$) is one-one
2. f(x) = cosx ($0 \leq x < \pi$) is one-one
3. f(x)= ax+b (-$\infty < x < \infty), a, b \in P$ is one-one
4. f(x) =e^{x^2} (-$\infty < x < \infty$) is not one-one
5. f(x) = x^2 (-$\infty < x < \infty$) is not one-one
6. f(x) = x^2 ($0 \leq x < \infty$) is one-one

Invers function

If function f: A$\longrightarrow$ B is one to one function , $f^{-1}: B \longrightarrow$ A is a function.

$f^{-1}[f(a)] = a, a \in A$

f[$f^{-1}(b)] = b , b \in B$

Equivalent sets

Two sets A and B are called equivalent sets If there exists one-one correspondence between A and B. It is denoted by A~B.

Property

1. Every set is quivalent to itself [A~A]
2. If A and B are equivalent sets, then B and A are equivalent sets.[A~B $\Rightarrow$ B~A]
3. If A and B are equivalent sets and B and C are equivalent sets then . A and C are equivalent sets, s[A~B and B~C $\Rightarrow$ A~C]

Countable (Denumerable)

The set A is said to be countable if A is equivalent to the set I of positive integers. (or) A is countable if exists a one-one function f from I onto A .

A = { f(1),f(2),f(3),… ,f(n),…. }

Example :

1. The set of integer is countable. { f(n) = $\frac{n-1}{2}$ (n = 1,3,5, ...) ,f(n) = $\frac{-n}{2}$ (n = 2,4,6,….}
2. The countable union of countable sets is countable.[If $A_1, A_2, \ldots .$ *are countable , then* $\bigcup_{n=1}^{\infty} A_n$ is countable]
3. The set of all rational numbers is countable.
4. An infinite set of real numbers that is countable is cardinally equivalent to the set N.
5. All finite sets are countable.
6. $E_n = \{\frac{0}{n}, \frac{-1}{n}, \frac{1}{n}, \frac{-2}{n}, \ldots . .$ } is countable ,then Q = $\bigcup_{n=1}^{\infty} E_n$ is countable.
7. Infinite subset of a countable set is countable.
8. The set of all rational numbers in [0, 1] is countable.
9. The set of all polynomial function with integer coefficient is countable.
10. The set of all polynomial function with rational coefficient is countable.
11. The set of all ordered pairs of integers is countable.
12. If A and B are countable sets,then the Cartesian product A$\times B$ is countable.
13. Let A be a countable set,then the set of all finite sequence from A is also countable.

Uncountable

An uncountable set is an infinit set which is not countable.

1. The set R of all real numbers is uncountable.
2. The set [0,1] = {x/0$\leq x \leq$ 1} is uncountable.
3. The set of all irrational numbers is uncountable.
4. The set of all characteristic function on I is uncountable
5. If B is a countable subset of the uncountable set A ,then A-B is uncountable.
6. If f : A$\longrightarrow$ B and the range of f is uncountable,then the domain of f is uncountable.

Binary expansion

The Binary expansion for a real number s uses only the digits '0' and '1'.

Example:

$0.a_1a_2a_3 \ldots. = \frac{a_1}{2} + \frac{a_2}{2^2} + \frac{a_3}{2^3} + \cdots.$

The ternary expansion.

The ternary expansion for a real number s uses only the digits '0' , '1' and '2'.

Example:

$0.a_1a_2a_3 \ldots. = \frac{a_1}{3} + \frac{a_2}{3^2} + \frac{a_3}{3^3} + \cdots.$

The cantor set

The cantor set K is the set of all numbers x in [0 , 1] which have a ternary expansion without the digit '1'.

Divide the closed interval [0,1] into three equalparts and remove the open middle interval $(\frac{1}{3}, \frac{2}{3})$ Again devide the each of the remaining two intervals into three equal parts and remove the middle part $(\frac{1}{3^2}, \frac{2}{3^2})$ and $(\frac{7}{9}, \frac{8}{9})$

Proceeding same proceeding we have
2^{r-1} *open intervals are removed each of the lenght*

$\frac{1}{3^r}$ .the remaining set constitutes cantor set or cantor ternary set.

The cantor set contains the points $0, 1, \frac{1}{3}, \frac{2}{3}, \frac{1}{9}, \frac{2}{9}, \frac{7}{9},$

- The cantor set is non-empty set.
- The cantor set is equivalent to [0,1]
- The cantor set is not countable

The Real Numbers

Real numbers R includes all of N, Z, and Q and also many new numbers, such as $\sqrt{2}$, e, and π *are irrational numbers*

Properties of real numbers

The set of real numbers R has two operations, called addition "+" and multiplication " · " and that these operations satisfy the field axioms.

1. For any $a, b \in$ R there is a number $a + b \in$ R and $a + b = b + a$.
2. For any $a, b, c \in$ R the identity $(a + b) + c = a + (b + c)$ is true.
3. There is a unique number $0 \in$ R so that, for all $a \in$ R, $a + 0 = 0 + a = a$.
4. For any number $a \in$ R there is a corresponding number denoted by $-a$with the property that $a + (-a) = 0$.

Under multiplication

1. For any $a, b \in$ R there is a number $ab \in$ R and $ab = ba$.
2. For any $a, b, c \in$ R the identity $(ab)c = a(bc)$ is true.
3. There is a unique number $1 \in$ R so that $a1 = 1a = a$ for all $a \in$ R.
4. For any number $a \in$ R, $a \neq 0$, there is a corresponding number denoted a^{-1} with the property that $aa^{-1} = 1$.
5. For any $a, b, c \in$ R the identity$(a + b)c = ac + bc$ is true.

Order Structure

The real number system also order structure.

$x < y$ or $x \leq y$.

The real number system is an ordered field, satisfying the four additional axioms.Here $a < b$ is now a statement that is either true or false.(Before $a + b$ and $a \cdot b$ were not statements, but elements of R.)

1. For any $a, b \in$ R exactly one of the statements $a = b$, $a < b$ or $b < a$ is true.
2. For any $a, b, c \in$ R if $a < b$ is true and $b < c$ is true, then $a < c$ is true.
3. For any $a, b \in$ R if $a < b$ is true, then $a + c < b + c$ is also true for any $c \in$ R.
4. For any $a, b \in$ R if $a < b$ is true, then $a \cdot c < b \cdot c$ is also true for any $c \in$ R for which $c > 0$.

Bounded above

The set A⊂R is said to be bounded above if there is a number NϵR such that x≤ N for all xϵA

Bounded below

The set A⊂R is said to be bounded above if there is a number MϵR such that M ≤ x for all xϵA

Bounded

If A is both bounded above and bounded below,then A is said to be bounded.

Ex: [0,1] is bounded set.

Upper Bounds

Let A be a set of real numbers.A number N is said to be an upper bound for A if x ≤ N for all x ∈ A.

Lower Bounds

Let A be a set of real numbers.A number M is said to be a lower bound for E if M ≤ x for all x ∈ A.

Maximum

Let A be a set of real numbers.If there is a number M that belongs to A and is larger than every other member of A,then M is called the maximum of the set E and we write M = maxA.

Minimum

Let A be a set of real numbers.If there is a number m that belongs to A and is smaller than every other member of A, then m is called the minimum of the set A and we write m = minA.

Example

The interval [0, 1] = {x : 0 ≤ x ≤ 1}
The maximum is 1
An upper bound is 1.
The minimum is 0
A lower bound is 0

Example

The interval (0, 1) = {x : 0 < x < 1}
has no maximum and no minimum.
An upper bound = 1
A lower bounds =0 but do not belong to the set.

The set of natural numbers N has no upper bound

Least Upper Bound/Supremum

Let A be a set of real numbers that is bounded above and nonempty.If L is the least of all the upper bounds, then L is said to be the least upper bound of A or the supremum of A and we write L = supA.

The least upper bound of a set A, if it exists, is unique.

Completeness Axiom

A nonempty set of real numbers that is bounded above has a least upper bound (i.e., if E is nonempty and bounded above, then sup$E$ exists and is a real number).

Greatest Lower Bound/Infimum

Let A be a set of real numbers that is bounded below and nonempty.If 'l' is the greatest of all the lower bounds of A, then 'l' is said to be the greatest lower bound of E or the infimum of A and we write l = inf A.

Example

The open interval (0, 1) has no maximum, but many upper bounds.Certainly 2 is an upper bound and so is 1.The least of all the upper bounds is the number 1.

Example

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E = φ or for unbounded sets.Th us we write

1. inf $\varphi = \infty$ and sup $\varphi = -\infty$.
2. If E is unbounded above, then supE = ∞.
3. If E is unbounded below, then inf E = −∞.
4. A set of real numbers E is bounded if and only if there is a positive number r so that |x| < r for all x ∈ E.
5. Find l.u.b and g.l.b of following

(i) (7,8)

l.u.b =8. g.l.b = 7

(ii) $\{\pi+1, \pi+2, \pi+3, \ldots\ldots.\}$

l.u.b = ∞ , g.l.b = $\pi + 1$

(iii) $\{\pi+1, \pi+\frac{1}{2}, \pi+\frac{1}{3}, \ldots\ldots\}$

l.u.b = $\pi + 1$, g.l.b = π

Find sup$E$ and inf E and (where possible) max$E$ and min$E$ for the following examples of sets:

(a) E = N
(b) E = Z
(c) E = Q
(d) E = R

(e) $E = \{-3, 2, 5, 7\}$
(f) $E = \{x : x^2 < 2\}$
(g) $E = \{x : x2 - x - 1 < 0\}$
(h) $E = \{1/n : n \in \mathbb{N}\}$
(i) $E = \{\sqrt{n} : n \in \mathbb{N}\}$

- If A is any nonempty subset of R that is bounded above, then A has a least upper bound in R.

- If A is any nonempty subset of R that is bounded below, then A has a greatest lower bound in R.
- If A is a non empty bounded subset of R and B is the set of all upper bounds for A, then $\text{g.l.b}_{y\in B}\, y = \text{g.l.b}_{x\in A}\, x$
- for every nonempty, finite set E that sup$E$ = max$E$.

Limits of sequence

Let $\{s_n\}_{n=1}^{\infty}$ be a sequence of real number L is called the limit of sequence $\{s_n\}$ if for every $\epsilon > 0$ there is a positive integer N such that $|s_n - L| < \epsilon$ for all n ≥N
That is, $\lim_{n\to\infty} s_n = L$ (or) $s_n \to L \text{ as } n \to \infty$

Uniqueness of Limits

Suppose that Lim $n\to\infty s_n = L_1$ and $\lim n\to\infty s_n = L_2$ are both true. Then $L_1 = L_2$.

Theorems

If $\lim_{n\to\infty} s_n = L$ and $\lim_{n\to\infty} t_n = M$,then

(i) $\lim_{n\to\infty}(s_n + t_n) = \lim_{n\to\infty} s_n + \lim_{n\to\infty} t_n = L+M$
(ii) $\lim_{n\to\infty}(s_n - t_n) = \lim_{n\to\infty} s_n - \lim_{n\to\infty} t_n = L-M$
(iii) $\lim_{n\to\infty}(s_n . t_n) = \lim_{n\to\infty} s_n . \lim_{n\to\infty} t_n = L.M$
(iv) $\lim_{n\to\infty}(s_n / t_n) = \lim_{n\to\infty} s_n / \lim_{n\to\infty} t_n = L/M$ $(M \neq 0)$
(v) If $s_n \leq t_n$,then $\lim_{n\to\infty} s_n \leq \lim_{n\to\infty} t_n$
(vi) $\lim_{n\to\infty} n\, s_n = nL$
(vii) $\lim_{n\to\infty} 1/ s_n = 1/L$ $(L\neq 0)$
(viii) $\lim_{n\to\infty}(-s_n) = -L$
(ix) $\lim_{n\to\infty} \sqrt{s_n} = \sqrt{L}$
(x) $\lim_{n\to\infty} {s_n}^2 = L^2$

Oscillatory Sequences

A sequence is said to **oscillate finitely**, if (i) It is bounded (ii) It is neither convergence nor divergence

A sequence is said to **oscillate infinitely**, if (i) It is not bounded (ii) It is neither convergence nor divergence

1. The sequence $\{(-1)^n\}$ is oscillates finitely.
2. The sequence $\{(-1)^n . n\}$ is oscillates infinitely.
3. If $\{s_n\}_{n=1}^{\infty}$ is converges to $L\neq 0$,then $\{(-1)^n s_n\}_{n=1}^{\infty}$ is oscillates.
4. If $\{s_n\}_{n=1}^{\infty}$ is diverges to infinity,then $\{(-1)^n s_n\}_{n=1}^{\infty}$ is oscillates.

Convergent and divergent sequence

A sequence that converges is said to be *convergent*.A sequence that fails to converge is said to diverge

- If $\{s_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ are sequences of real numbers that diverge to infinity,then

$\{s_n + t_n\}_{n=1}^{\infty}$ and $\{s_n t_n\}_{n=1}^{\infty}$ diverge to infinity.

- If $\{s_n\}_{n=1}^{\infty}$ *diverges to infinity* and $\{t_n\}_{n=1}^{\infty}$ converge, then $\{s_n + t_n\}_{n=1}^{\infty}$ diverge to infinity.

Ex:

1. $\{\frac{1}{n}\}_{n=1}^{\infty}$ is has limit '0'
2. $\{n\}_{n=1}^{\infty}$ is does not has limit
3. $\{1\}_{n=1}^{\infty}$ is has limit '1'
4. $\{\frac{2n}{n+3}\}_{n=1}^{\infty}$ is has limit '2'
5. $\{\frac{1}{\sqrt{n+1}}\}_{n=1}^{\infty}$ is has limit '0'
6. $\{\frac{n^2}{n+5}\}_{n=1}^{\infty}$ is has limit '0'
7. $\{\frac{3n}{n+7n^2}\}_{n=1}^{\infty}$ is has no limit
8. $\{\frac{10^7}{n}\}_{n=1}^{\infty}$ is has limit '0'
9. $\{n - \frac{1}{n}\}_{n=1}^{\infty}$ is has no limit
10. $\lim_{n\to\infty}(1+\frac{1}{n})^n$ = e (2<e≤3)
11. $\lim_{n\to\infty}\frac{n^2}{2n^2+1} = \frac{1}{2}$
12. $\lim_{n\to\infty} n$ does not exist.
13. $\lim_{n\to\infty}(-1)^n$ does not exist.
14. $\lim_{n\to\infty}\frac{1^2+2^2+3^2+\cdots n^2}{n^3} = \frac{1}{2}$
15. $\lim_{n\to\infty}\frac{1\ \ +2\ \ +3\ \ +\cdots n}{n^2} = \frac{1}{3}$
16. $\lim_{n\to\infty} n^2 = \infty$
17. $\lim_{n\to\infty}\frac{n^3+1}{n^2+1} = \infty$
18. if $s_n \to\infty$ then $-s_n \to -\infty$.
19. if $s_n \to\infty$ then $(s_n)^2 \to\infty$
20. $\lim_{n\to\infty} s_n = 0$.Sho w that $\lim_{n\to\infty} 1/s_n = \infty$.the converse not true
21. if $x_n \to \infty$ then the sequence $s_n = \frac{x_n}{x_n+1}$ is convergent. the converse not true
22. $\lim_{n\to\infty}\frac{2n}{n+4n^{\frac{1}{2}}} = 2$
23. If P is a polynomial function of third degree p(x) = ax^3+bx^2+cx+d (a,b,c,d∈ R)

    Then $\lim_{n\to\infty}\frac{p(n+1)}{(n)} = 1$

Theorem

- If $\{s_n\}_{n=1}^{\infty}$ is a sequence of non-negative numbers and $\lim_{n\to\infty} s_n$ = L,than L≥ 0

- If $\{s_n\}_{n=1}^{\infty}$ is a sequence of non-negative numbers with $s_n \le M (n \in I)$and $\lim_{n\to\infty} s_n$ = L,than L≤ MBounded sequence
- $\{s_n\}_{n=1}^{\infty}$ is bounded iff there exists M∈ R such that $|s_n| \le M\ (n \in I)$

- If the sequence of real number $\{s_n\}_{n=1}^{\infty}$ is convergent, then $\{s_n\}_{n=1}^{\infty}$ is bounded.
- Every convergent sequence is bounded.

Limit superior

Let $\{s_n\}_{n=1}^{\infty}$ be a sequence of real numbers that is bounded above, and let $M_n = l.u,b\{s_n, s_{n+1}, s_{n+2}, \dots \}$

(a) If $\{M_n\}_{n=1}^{\infty}$ is converges and define $\limsup_{n\to\infty} s_n = \lim_{n\to\infty} M_n$

(b) If $\{M_n\}_{n=1}^{\infty}$ is divergence to - ∞ then $\limsup_{n\to\infty} s_n$ =- ∞

Definition

If $\{s_n\}_{n=1}^{\infty}$ is a sequence of real numbers that is not bounded above, then $\limsup_{n\to\infty} s_n = \infty$

Theorem

If $\{s_n\}_{n=1}^{\infty}$ is a convergence sequence of real numbers then $\limsup_{n\to\infty} s_n = \lim_{n\to\infty} s_n$

Limit inferior

Let $\{s_n\}_{n=1}^{\infty}$ be a sequence of real numbers that is bounded below, and let $m_n = g.l,b\{s_n, s_{n+1}, s_{n+2}, \dots \}$

(a) If $\{m_n\}_{n=1}^{\infty}$ is converges and define $\liminf_{n\to\infty} s_n = \lim_{n\to\infty} m_n$

(b) If $\{m_n\}_{n=1}^{\infty}$ is divergence to ∞ then $\liminf_{n\to\infty} s_n = \infty$

Ex:

1. If $s_n = (-1)^n$,Then, $\limsup_{n\to\infty} s_n$= 1 and $\liminf_{n\to\infty} s_n$ = -1
2. If $s_n = -n\ (n \in I)$,Then, $\limsup_{n\to\infty} s_n$= -∞

Definition

If $\{s_n\}_{n=1}^{\infty}$ is a sequence of real numbers that is not bounded below Then, $\liminf_{n\to\infty} s_n = -\infty$

Theorem

If $\{s_n\}_{n=1}^{\infty}$ is a convergence sequence of real numbers then $\liminf_{n\to\infty} s_n = \lim_{n\to\infty} s_n$

Theorem

- If $\{s_n\}_{n=1}^{\infty}$ is a sequence of real numbers and $\limsup_{n\to\infty} s_n = \liminf_{n\to\infty} s_n$ = L ,then

  $\lim_{n\to\infty} s_n$ = L
- If $\{s_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ are bounded sequence of real numbers and if $s_n \le t_n\ (n \in I)$,then

  $\limsup_{n\to\infty} s_n \le \limsup_{n\to\infty} t_n$

  $\text{lliminf}_{n\to\infty} s_n \le \liminf_{n\to\infty} t_n$
- If $\{s_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ are bounded sequence of real numbers ,then

  $\limsup_{n\to\infty}(s_n + t_n) \le \limsup_{n\to\infty} s_n + \limsup_{n\to\infty} t_n$

  $\liminf_{n\to\infty}(s_n + t_n) \ge \liminf_{n\to\infty} s_n + \liminf_{n\to\infty} t_n$

Cauchy sequence

If $\{s_n\}_{n=1}^{\infty}$ is a sequence of real numbers then $\{s_n\}_{n=1}^{\infty}$is called a Cauchy sequence if for any ∈> 0 there exists N∈I such that $|s_m - s_n| <$∈ (m,n∈ N)

Theorem

- If the sequence of real numbers $\{s_n\}_{n=1}^{\infty}$ is convergence ,then $\{s_n\}_{n=1}^{\infty}$ is Cauchy sequence
- If $\{s_n\}_{n=1}^{\infty}$ is Cauchy sequence of real number,then $\{s_n\}_{n=1}^{\infty}$ is bounded.
- If $\{s_n\}_{n=1}^{\infty}$ is Cauchy sequence of real number,then $\{s_n\}_{n=1}^{\infty}$ is convergence.

Summability of sequence

Let $\{s_n\}_{n=1}^{\infty}$ be a sequence of real numbers and let $\sigma_n = \frac{s_1+s_2+s_3+. \ . \ .+s_n}{n}$ (n$\in N$)

$\{s_n\}_{n=1}^{\infty}$ is summable to L(C,l) If the sequence $\{\sigma_n\}_{n=1}^{\infty}$ conveges to L.

This case we write $\lim_{n\to\infty} s_n = L$ (C,l)

Theorem (Cauchy's First theorem on limits)

If $\lim_{n\to\infty} s_n = L$,then $\lim_{n\to\infty} s_n = L$ (C,l)

- A sequence that diverge to infinitly cannot be (C,l) summable.

Subsequence

Let $\{s_n\}_{n=1}^{\infty}$ be any sequence.Then by a subsequence of this sequence we mean any Sequence s_{n1}, s_{n2}, s_{n3}, s_{n4}, . . . where $n_1 < n_2 < n_3 <$ . . .is an increasing sequence of natural numbers.

Every sequence contains a monotonic subsequence

NATIONAL ACADEMY DHARMAPURI

REAL ANALYSIS – Test -1

[Cardinal numbers – countable set – uncountable – bounded set]

1. What is the number of proper subset of given finite set with n elements ?
   (a) 2n-1 (b) 2n-2 (c) $2^n - 1$ **(d) $2^n - 2$**
2. If n(A) =3,n(B) =6 and A⊆ B,then the number of elements in n(A∪ B) is equal to
   (a) 3 (b) 9 **(c) 6** (d) 12
3. Let X bethe set of all nonempty finite subsets of N.Which one of the following is not an equivalence relation on X.
   (a) A~ B if and only if min A = min B
   (b) A~ B if and only if A,B have same number of elements
   (c) A~ B if and only if A = B
   (d)A~ B if and only if A ∩ B =φ
4. The function e^x from R to R is
   (a) **Both one-one and onto** (b) One-one but not onto
   (c) Onto but one-one (d) Neither one-one nor onto
5. $|a-b| = |c-d|$,then
   (a) a = b+c-d (b) a = b-c+d
   (c) a = b+c-d and a = b-c+d **(d) a = b+c-d or a = b-c+d**
6. Let x,y∈R,If $|x+y| = |x| + |y|$ then
   (a) $|x-y| = |x| - |y|$ (b) $|xy|$ = xy
   (c) $|x^2+y| = |x^2| + |y|$ **(d) $|x+y| = x+y$**
7. The set of all real numbers x for which there is some positive real number y such that x< y is equal to
   (a) R (b) the set of all negative real numbers
   (b) {0} (d) the empty set.
8. Let x∈ $R - \{0\}$ then the correct statement is.
   (a) If $x^2 \in Q, then\ x^3 \in Q$ (b) If $x^3 \in Q, then\ x^2 \in Q$
   (c) If $x^2 \in Q\ and$If $x^4 \in Q, then\ x^3 \in Q$ (d) **If $x^2 \in Q\ and$If $x^5 \in Q, then\ x \in Q$**
9. The set of all rational numbers is
   (a) finite (b) countably infinite **(c) countable** (d) uncountable
10. Let A be a subset of real numbers containing all the rational numbers.which of the following ststements is true?
    (a) **A is countable** (b) If A is uncountable,then A = R
    (c) If A is open, then A =R (d)None of the above statement is true.
11. Every infinite set has a
    (a) countable sub set (b) uncountable subset
    (c) countable and uncountable both sets (d) none of these
12. The union of a finite or countable collection of countable set is
    (a) countable (b) uncountable (c) infinite (c) none of these

13. The interval [0,1] is

(a) Countable **(b) uncountable** (c) finite (d) at most countable

14. The set of irrational numbers is

(a) Countable **(b) uncountable** (c) finite (d) nowhere dense in R

15. The set of all real numbers R is

(a) unbounded (b) bounded from below

(c) bounded from above (d) bounded

16. The closed interval [0,1] is

(a) bounded above (b) unbounded below

(c) unbounded above (d) no maximal element

17. The set of natural numbers has

(a) upper bound **(b) lower bound** (c) maximal element (d) none of these

18. The set R+ = $(0,\infty)$ is

(a) bounded above **(b) bounded below**

(c) unbounded below (d) none bof these

19. The sequence $\{(-1)^n(1+\frac{1}{n})\} is$

(a) Bounded below but not bounded above. (b) Bounded above but not bounded below

(b) Bouned . **(b) Not bounded.**

20. Let A,B ⊂ R and C = {a+b/a$\in A, b \in B$}. then the false statement in the following is.

(a) If A,B aer bounded sets,then C is bounded set

(b)If C is bounded set,then A,B are bounded sets.

(c) If R-A,R-B are bounded sets,then R-C is a bounded set

(d)If R-C is a bounded set,then R-A,R-B are bounded sets

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