

Test no : 1 (Complex numbers)

1. The amplitude of a complex number i is

- a) 0 b) $\pi/2$ c) π d) 2π

2. The amplitude of the quotient of two complex numbers is

- a) the sum of their amplitudes b) the difference of their amplitudes
c) the product of their amplitudes d) the quotient of their amplitudes

3. The $z = x + iy$ then $\bar{z} =$

- a) $x^2 + y^2$ b) $x^2 + iy^2$ c) $x^2 - y^2$ d) imaginary

4. If z_1 & z_2 are any two complex numbers, then $\frac{|z_1 + z_2|}{|z_1 z_2|}$ is

- a) $\frac{|z_1 + z_2|}{|z_2|}$ b) $\frac{|z_1 + z_2|}{|z_1|}$ c) $\frac{|z_1 + z_2|}{|z_2||z_1|}$ d) $\frac{|1|}{|z_1|} \frac{|1|}{|z_2|}$

5. $\lim_{z \rightarrow 2i} (2x + iy^2)^2 =$

- a) $8i$ b) $-8i$ c) 16 d) $4i$

6. The equation $|z + 4i| + |z - 4i| = 10$ represents

- a) Circle b) ellipse c) parabola d) hyperbola

7. The equation $|z + 5i| = |z - 5i|$ represents

- a) Circle b) ellipse c) parabola d) Real axis

8. The equation $|z + 1| = \sqrt{2}|z - 1|$ represents

- a) Circle b) ellipse c) parabola d) hyperbola

9. The equation $|z + 2| + |z - 2| = 4$ represents

- a) Circle b) a st. line c) parabola d) hyperbola

10. The equation $|2z - 1| = |z - 2|$ represents

- a) Circle b) a st. line c) parabola d) hyperbola

11. The equation $|z - 1|^2 + |z - 2|^2 = 4$ represents

- a) Circle b) a st. line c) parabola d) hyperbola

12. The equation $\arg\left(\frac{z-a}{z-b}\right) = k$ represents

- a) Circle b) a st. line c) parabola d) hyperbola

13. The equation $\arg\left(\frac{z-1}{z+1}\right) = \pi/3$ represents

- a) Circle b) a st. line c) parabola d) hyperbola

14. Let $f(z) = \begin{cases} \frac{Re z}{|z|} & , z \neq 0 \\ 0 & , z = 0 \end{cases}$ and $g(z) = \begin{cases} \frac{x^3(1+i)-y^3(1-i)}{x^2+y^2} & , z \neq 0 \\ 0 & , z = 0 \end{cases}$

- a) both $f(z)$ & $g(z)$ are continues at the origin
 b) both $f(z)$ & $g(z)$ are not continues at the origin
 c) $f(z)$ continues at the origin & $g(z)$ is not continues at the origin
 d) $f(z)$ is not continues at the origin & $g(z)$ is continues at the origin

15. The multiplicative identity in \mathbb{C} is

- a) (0,0) b) (1,1) c) (0,1) d) (1,0)

16. $\left(\frac{1+\cos\frac{\pi}{8}+isin\frac{\pi}{8}}{1+\cos\frac{\pi}{8}-isin\frac{\pi}{8}} \right)^8 =$

- a) 1+i b) 1-i c) 1 d) -1

17. If α is a complex number such that $\alpha^2 + \alpha + 1 = 0$ then $\alpha^{31} =$

- a) α b) α^2 c) 0 d) 1

18. If p is a complex number & $|q| = 1$ then $\left| \frac{p-q}{pq-1} \right|$ is

- a) i b) -i c) 1 d) -1

19. $\lim_{z \rightarrow 2} \left(\frac{z^2-4}{z-2} \right) =$

- a) 2 b) 4 c) 1 d) 8

20. If z_1 & z_2 are the image of the complex plane of two diametrically opposite point on the Riemann's sphere then $\overline{z_1 z_2}$ is

- a) 0 b) 1 c) -1 d) i

Answer key :

1	6	11	16	21
2	7	12	17	22
3	8	13	18	23
4	9	14	19	24
5	10	15	20	25

Test no : 2 (Analytic functions)

1. The function $w = \frac{z^2+1}{z^2-1}$ is analytic

- a) for all z b) for all z except $1, -1$ c) for all z except $i, -i$ d) for all z except $1 + 2i$

2. Which of the following is not an entire function ?

- a) e^z b) e^{-z} c) $z^2 - 1$ d) $\log z$

3. Which of the following is called C-R equation for the function $f(z) = u + iv$ to be analytic at z ?

- a) $u_x = -v_y$ & $u_y = v_x$ b) $u_x = v_y$ & $u_y = -v_x$ c) $u_x = v_y$ & $u_y = v_x$ d) $u_x = -v_x$ & $u_y = v_y$

4. Which of the following is called C-R equation for polar coordinates ?

- a) $u_r = v_\theta$ & $u_\theta = -v_r$ b) $u_r = \frac{1}{r}v_\theta$ & $\frac{1}{r}u_\theta = v_r$
c) $u_r = \frac{1}{r}v_\theta$ & $\frac{1}{r}u_\theta = -v_r$ d) $\frac{1}{r}u_r = v_\theta$ & $\frac{1}{r}u_\theta = v_r$

5. Define $f(z) = \frac{\sin z}{z}$ $z \neq 0$ then

- a) $\lim_{z \rightarrow 0} f(z)$ does not exist b) $f(z)$ is continuous at $z=0$
b) $f(z)$ does not continuous at $z=0$ d) $\lim_{z \rightarrow 0} f(z) = 1$

6. For the function $f(z) = u + iv$ to be differentiable

- a) necessary & sufficient b) necessary & not sufficient
b) not necessary & sufficient d) none

12. For the function $f(z) = u + iv$ to be differentiable, which of the following is true

- a) u_x, v_y, u_y, v_x exists b) $u_x = v_y$ & $u_y = -v_x$ holds
b) u_x, v_y, u_y, v_x exists & continuous d) all

7. Let $f(z) = \begin{cases} \frac{\sin z}{z} & , z \neq 0 \\ 1 & , z = 0 \end{cases}$ then $f(z)$ at $z=0$

- a) $f(z)$ continuous b) $f(z)$ not continuous c) $f(z)$ is well d) none

8. For the function $f(z)$ a point z_0 is said to be singular point if

- a) $f(z)$ is analytic at z_0 b) $f(z)$ is not analytic at z_0 c) $f(z)$ is continuous at z_0 n) none

9. For the function $f(z) = \frac{(z+2)^3}{(z-2)(z-1)^2}$ which of the following is true

- a) $z = -2$ is a zero of order 3 b) $z = 1$ is a pole of order 2 c) $z = 2$ is a pole of order d) all

10. For the function $f(z) = \bar{z}$ is

- a) entire function b) differentiable at $z=2$ c) $z=2$ is a pole of order d) all
11. If $f(z) = a(x^2 - y^2) + ibxy + c$ is differentiable at every point, the constants
- a) $2b=a$ b) $4b=a$ c) $2a=b$ d) $a=b$
12. The function $f(z) = |z|^2$ is
- a) differentiable at $z=0$ b) nowhere differentiable c) not harmonic d) differentiable at $z \neq 0$
13. If $f(z) = u + iv$ is analytic & $u = 2x(1 - y)$ then $f(z) =$
- a) $2z + iz^2$ b) $2z - iz^2$ c) $z + iz^2/2$ d) $z - iz^2/2$
14. If $f(z) = u + iv$ is analytic & $v = 2xy$ then $f(z) =$
- a) $z^4 + c$ b) $z^2 + c$ c) $z + c$ d) $z - iz^2/2$
15. If $f(z) = (x^2 + ay^2 - 2xy) + i(bx^2 - y^2 + 2xy)$ is analytic, then the value of a, b are respectively
- a) -1,1 b) -1,2 c) 1,-1 d) 2,-1
16. If $f(z) = a(x^2 - y^2) + ibxy + c$ is differentiable at every point, the constants
- a) $2b=a$ b) $4b=a$ c) $2a=b$ d) $a=b$
17. If $u = 2x + x^2 - my^2$ is harmonic, then the value of m is
- a) 3 b) 2 c) 0 d) 1
18. If the function $f(z) = \frac{z}{1+z}$, $u + iv =$
- a) $\frac{x}{x^2+y^2} - \frac{iy}{x^2+y^2}$ b) $\frac{x^2+y^2+x}{(x+1)^2+y^2} + \frac{iy}{(x+1)^2+y^2}$ c) $\frac{x^2+y^2}{(x+1)^2+y^2} + \frac{iy}{(x+1)^2+y^2}$ d) $(x^2 + 5) + 7yi$
19. Any two harmonic conjugates of a given harmonic function $u(x,y)$ differ by
- a) x b) y c) xy d) constant
20. The image of the strip $0 < x < 1$ under the transformation $w = iz$ is
- a) $0 < u < 1$ b) $0 < v < 1$ c) $1 < u < 2$ d) $1 < v < 2$

Answer key :

1	6	11	16	21
2	7	12	17	22
3	8	13	18	23
4	9	14	19	24
5	10	15	20	25

Test no : 3 (Radius of convergent)

1. The radius of convergence of $\sum \left(\frac{z^n}{n!}\right)$ is
a) ∞ b) e c) $1/e$ d) 1
2. The radius of convergence of $\sum \left(\frac{n}{n+1}\right) z^n$ is
a) ∞ b) e c) $1/e$ d) 1
3. The radius of convergence of $\sum \left(\frac{n+1}{(n+2)(n+3)}\right) z^n$ is
a) ∞ b) e c) $1/e$ d) 1
4. The radius of convergence of $\sum \left(\frac{n!^2}{2n!}\right) z^n$ is
a) ∞ b) e c) $1/e$ d) 4
5. The radius of convergence of $\sum \left(\frac{(-1)^n}{n}\right) (z - 2i)^n$ is
a) ∞ b) e c) $1/e$ d) 1
6. The radius of convergence of $\sum (3 + 4i) z^n$ is
a) ∞ b) e c) $1/5$ d) 4
7. The radius of convergence of $\sum \left(1 + \frac{1}{n}\right)^{n^2}$ is
a) ∞ b) e c) $1/e$ d) 0
8. The invariant points of $w = \frac{z-15}{z-7}$ is
a) 3,-5 b) 3,-3 c) 3,5 d) -3,-5
9. The invariant points of $w = \frac{3z-4}{z-1}$ is
a) 2, -2 b) 0, 1 c) $\pm i$ d) ± 3
10. The invariant points of $w = \frac{z}{2-z}$ is
a) 2, -2 b) 0, 1 c) $\pm i$ d) ± 3
11. The invariant points of $w = \frac{z-1}{z+1}$ is
a) 2, -2 b) 0, 1 c) $\pm i$ d) ± 3
12. The invariant points of $w = \frac{z}{2z-1}$ is

- a) 2, -2 b) 0, 1 c) $\pm i$ d) ± 3

13. The invariant points of $w = \frac{3z-5}{z+1}$ is

- a) 2, -2 b) 0, 1 c) $1 \pm 2i$ d) ± 3

14. The invariant points of $w = \frac{z+1}{2z+1}$ is

- a) $\pm \frac{1}{\sqrt{2}}$ b) $\pm \frac{1}{\sqrt{3}}$ c) $\pm \frac{1}{2}$ d) $\pm \frac{1}{3}$

15. The transformation $w = z + c$ is said to be

- a) Translation b) Magnification c) Rotation d) Inversion

16. The transformation $w = az$ is said to be

- a) Translation b) Magnification & Rotation c) Inversion d) none

17. The transformation $w = 1/z$ is said to be

- a) Translation b) Magnification c) Rotation d) Inversion & Reflection

18. The curve & their images under $w = 1/z$ are given below, state which of the following is true

- 1) Circle not thro' 0 into circle not thro' 0 2) St. line not thro' 0 into circle thro' 0
3) Circle thro' 0 into St. line not thro' 0 4) St. line thro' 0 into St. line thro' 0

- a) 1,2,3 b) 2,3,4 c) 1,2,3,4 d) 3,4

19. The angle of rotation at $z = 1+i$ under the map $w = Z^2$ is

- a) 45 b) 0 c) 90 d) 180

20. The image of the circle $|z - 3i| = 3$ under the map $w = 1/z$ is

- a) circle b) st. line c) square d) rectangle

Answer key :

1	6	11	16	21
2	7	12	17	22
3	8	13	18	23
4	9	14	19	24
5	10	15	20	25

Test no : 4 (Transformations & complex integeation)

1. The image of the strip $0 < x < 1$ under the transformation $w = iz$ is
 - a) $0 < u < 1$
 - b) $0 < v < 1$
 - c) $1 < u < 2$
 - d) $1 < v < 2$
2. The image of the strip $0 < x < 1$ under the transformation $w = iz$ is
 - a) $v > 1$
 - b) $v = 1$
 - c) $0 < v < 1$
 - d) $-1 < v < 0$
3. The image of the strip $0 < x < 1$ under the transformation $u^2 + v^2 = \lambda x$ is
 - a) $u^2 + v^2 = 1/\lambda$
 - b) $u = 1/\lambda$
 - c) $v = 1/\lambda$
 - d) none
4. The image of the strip $0 < y < 1$ under the transformation $w = 1/z$ is
 - a) $u^2 + v^2 + v > 0$
 - b) $u^2 - v^2 - 2cv > 0$
 - c) $u^2 + v^2 > 0$
 - d) none
5. The image of the strip $0 < y < 1/2c$ under the transformation $w = 1/z$ is
 - a) $u^2 + v^2 + 2cv > 0$
 - b) $u^2 - v^2 - 2cv > 0$
 - c) $u^2 + v^2 > 0$
 - d) none
6. The transformation $w = 1/z$ transforms
 - a) circle not through the origin in the z - plane into the circle through the origin in the w -plane
 - b) circle through the origin in the z - plane into the st.line passing through the origin in the w -plane
 - c) st.line not through the origin in the z - plane into the circle through the origin in the w -plane
 - d) st.line through the origin in the z - plane into the st.line passing through the origin in the w -plane
7. The transformation $w = e^z$ transforms the imaginary axis in the Z axis into
 - a) a unit circle with center at origin in the W plane
 - b) a circle of radius 2 with center at origin in the W plane
 - c) a straight line in the W plane
 - d) an ellipse in the W plane
8. The bilinear transformation that map the points $z_1 = 0, z_2 = -1, z_3 = \infty$ into $w_1 = -1, w_2 = -2 - i, w_3 = i$ is
 - a) $w = \frac{2+iz}{z+2}$
 - b) $w = \frac{2+iz}{z-2}$
 - c) $w = \frac{-2+iz}{z+2}$
 - d) $w = \frac{-2+iz}{z-2}$
9. The bilinear transformation that map the points $z_1 = \infty, z_2 = i, z_3 = 0$ into $w_1 = 0, w_2 = i, w_3 = \infty$ is
 - a) $w = 1/z$
 - b) $w = -1/z$
 - c) $w = z$
 - d) $w = iz$
10. The bilinear transformation that map the points $z_1 = 0, z_2 = -i, z_3 = -1$ into $w_1 = i, w_2 = 1, w_3 = 0$ is

a) $w = (i) \frac{z+1}{z-1}$ b) $w = (-i) \frac{z+1}{z-1}$ c) $w = (i) \frac{z-1}{z+1}$ d) $w = (-i) \frac{z-1}{z+1}$

11. If $f(z)$ is an entire function & if $\int \frac{f(z)}{z-a} dz = 0$ then

- a) $z=a$ lies inside C b) $z=a$ lies on the boundary of C c) $z=a$ lies outside of C d) none

12. If $f(z)$ is analytic with in & on C & $z=a$ lies inside C, then $\int \frac{f(z)}{z-a} dz$

- a) 0 b) $2\pi i f'(a)$ c) $\pi i f(a)$ d) $2\pi i$

13. If $f(z)$ is analytic in a simple closed curve C, then $\int f(z) dz$ is

- a) 0 b) $2\pi i f'(a)$ c) $\pi i f(a)$ d) $2\pi i$

14. If C is the positively oriented circle $|z| = 2$, then $\int \frac{z}{z^2-1} dz$ is

- a) 0 b) $2\pi i$ c) $\pi i f(1)$ d) $-2\pi i$

15. If C is the unit circle $|z| = 1$, then $\int z^m (\bar{z})^{-n} dz$ is

- a) 0 b) $2\pi i mn$ c) $m + n$ d) $m - n$

16. If C is the unit circle $|z| = 1$, then $\int \bar{z} dz$ is

- a) 0 b) πi c) $2\pi i$ d) $3\pi i$

17. If C is a straight line from $z=0$ to $z=4+2i$, then $\int \bar{z} dz$ is

- a) 0 b) $10 + \frac{8i}{3}$ c) $10 - \frac{8i}{3}$ d) $8 + \frac{10i}{3}$

18. If C is the unit circle $|z| = 1$, then $\int z^2 dz$ is

- a) 0 b) $2\pi i$ c) π d) $3\pi i$

19. If C is the unit circle $|z| = 1$, then $\int z e^z dz$ is

- a) 0 b) $2\pi i$ c) π d) $3\pi i$

20. If C is the unit circle $|z| = 1$, then $\int z z^{-z} dz$ is

- a) 0 b) $2\pi i$ c) π d) $3\pi i$

Answer key :

1	6	11	16	21
2	7	12	17	22
3	8	13	18	23
4	9	14	19	24
5	10	15	20	25

Test no : 5 (complex Integrations)

1. If C is the unit circle $|z| = 1$, then $\int e^z \sin z \, dz$ is
a) 0 b) $2\pi i$ c) π d) $3\pi i$
2. If C is the unit circle $|z| = 1$, then $\int \frac{1}{z^2-4} \, dz$ is
a) 0 b) $2\pi i$ c) π d) $3\pi i$
3. If C is the unit circle $|z| = 1$, then $\int \frac{1}{z^2+2z+2} \, dz$ is
a) 0 b) $2\pi i$ c) π d) $3\pi i$
4. If C is the unit circle $|z| = 1$, then $\int \frac{z^2}{z-3} \, dz$ is
a) 0 b) $2\pi i$ c) π d) $3\pi i$
5. If C is the unit circle $|z| = 1$, then $\int \frac{z^2}{z-3} \, dz$ is
a) 0 b) $2\pi i$ c) π d) $3\pi i$
6. If C is the circle $|z - 2| = 3$, then $\int \frac{1}{z-1} \, dz$ is
a) 0 b) $2\pi i$ c) $2\pi i e^2$ d) $2\pi i e^{-2}$
7. If C is the circle $|z - 2| = 3$, then $\int \frac{1}{z} \, dz$ is
a) 0 b) $2\pi i$ c) $2\pi i e^2$ d) $2\pi i e^{-2}$
8. If C is the circle $|z| = 2$, then $\int \frac{e^z}{(z-2)^2} \, dz$ is
a) 0 b) $2\pi i$ c) $2\pi i e^2$ d) $-2\pi i e^{-1}$
9. If C is the circle $|z| = 2$, then $\int \frac{e^{-z}}{(z-\frac{\pi i}{2})^2} \, dz$ is
a) 0 b) 2π c) $2\pi i e^2$ d) $2\pi i e^{-2}$
10. If C is the circle $|z| = 2$, then $\int \frac{ze^{-z}}{(z-2)^2} \, dz$ is
a) 0 b) $-\pi i/2$ c) $2\pi i e^2$ d) $-2\pi i e^{-1}$
11. If C is the circle $|z| = 1$, then $\int \frac{z^2+5}{z-3} \, dz$ is
a) 0 b) $2\pi i$ c) $2\pi i$ d) $-28\pi i$

12. If C is the circle $|z + 1| = 1$, then $\int \frac{3z^2+7z+1}{z+1} dz$ is

- a) 0 b) $2\pi i$ c) $3\pi i$ d) $-6\pi i$

13. If C is the circle $|z| = 2$, then $\int \frac{3z^2+3z-5}{z-1} dz$ is

- a) 0 b) $2\pi i$ c) $3\pi i$ d) $-6\pi i$

26. If C is the circle $|z| = 1$, then $\int \frac{z^2+5z+6}{z-2} dz$ is

- a) 0 b) $20\pi i$ c) $2\pi i$ d) $40\pi i$

14. If C is $|z| = R < 1$ then $\int \frac{z^2+1}{z^2-1} dz$ is

- a) 0 b) $2\pi i$ c) πi d) $2\pi i$

15. If C is the circle $|z| = 1$, then $\int \frac{e^z}{z^5} dz$ is

- a) $2\pi i/5$ b) $\pi i/12$ c) $2\pi i e^2$ d) $2\pi i e^{-2}$

16. If C is the circle $|z| = 3$, then $\int \frac{ze^{-z}}{(z-2)^2} dz$ is

- a) $2\pi i/5$ b) $\pi i/12$ c) $2\pi i e^2$ d) $-2\pi i e^{-2}$

17. If C is the circle $|z| = 1$, then $\int \frac{e^{iz}}{z^3} dz$ is

- a) $2\pi i/5$ b) $\pi i/12$ c) $2\pi i e^2$ d) $-\pi i$

18. If C is the circle $|z| = 2$, then $\int \frac{e^z}{(z-2)^4} dz$ is

- a) $2\pi i/5$ b) $\pi i e/3$ c) $2\pi i e^2$ d) $-\pi i$

19. If C is a closed curve enclosing the origin, then the value of $\int \frac{1}{z^2 e^z} dz$ is

- a) 0 b) $-2\pi i$ c) $2\pi i$ d) $4\pi i$

20. If C is $|z| = 1$ then $\int \frac{e^z}{z^n} dz$ is

- a) 0 b) $2\pi i$ c) πi d) $\frac{2\pi i}{(n-1)!}$

Answer key :

1	6	11	16	21
2	7	12	17	22
3	8	13	18	23
4	9	14	19	24

5	10	15	20	25
---	----	----	----	----

Test no : 6 (Residues)

1. If C is the positively oriented circle $|z - i| = 2$, then $\int \frac{e^z}{z^2+4} dz$ is

- a) πe^{2i} b) $\frac{\pi}{2}(e^{2i} - e^{-2i})$ c) $\frac{\pi}{2}(e^{2i})$ d) $2\pi e^{2i}$

2. The poles of $z \cot z$ are

- a) $2\pi n$ b) $n\pi$ c) $(2n + 1)\pi$ d) $(2n - 1)\pi$

3. The residue of $f(z) = \frac{z+1}{2(z-2)}$ at $z=2$ is

- a) $1/5$ b) $1/4$ c) $1/3$ d) $3/2$

4. The residue of $f(z) = \frac{4}{z^3(z-2)}$ at a simple pole is

- a) $1/5$ b) $1/4$ c) $1/3$ d) $1/2$

5. The residue of $f(z) = z \cot \frac{1}{z}$ at $z=0$ is

- a) $1/5$ b) $1/4$ c) $1/3$ d) $-1/2$

6. The residue of $f(z) = \tan z$ at $z = \pi/2$ is

- a) $1/5$ b) $1/4$ c) $1/3$ d) -1

7. The residue of $f(z) = \frac{z+1}{z^2-2z}$ are

- a) $1/2$ b) $-1/2$ c) $3/2$ d) $-3/2$

8. The residue of $f(z) = \frac{3}{e^z-1}$ at $z=0$ is

- a) $1/5$ b) $1/4$ c) $1/3$ d) 3

9. The residue of $f(z) = \frac{z+1}{z^2(z-2)}$ at $z=0$ is

- a) $1/5$ b) $1/4$ c) $1/3$ d) $-3/4$

10. The residue of $f(z) = \frac{1+2z}{z^2-z-2}$ at $z = 2$ is

- a) $1/5$ b) $1/4$ c) $5/3$ d) -1

11. The residue of $f(z) = \frac{\sin z}{(z-1)^2}$ at $z = 1$ is

- a) $\cos 1$ b) $\sin 1$ c) $1/3$ d) -1

12. The residue of $f(z) = ze^{1/z}$ at $z = 0$ is

- a) 1/5 b) 1/4 c) 1/3 d) 1/2

13. If C is the circle $|z| = 3/2$, then $\int \frac{\cos\pi z^2 + \sin\pi z^2}{(z-1)(z-2)} dz$ is

- a) 0 b) $2\pi i$ c) $2\pi i e^2$ d) $-2\pi i e^{-1}$

14. If C is the circle $|z| = 2$, then $\int \frac{e^{3z}}{(z+1)^4} dz$ is

- a) 0 b) $-\pi i/2$ c) $9\pi i/e^3$ d) $-2\pi i e^{-1}$

15. If C is the circle $|z - 1| = 2$, then $\int \frac{z+3z^3}{1-z^2} dz$ is

- a) 0 b) $-4\pi i$ c) $2\pi i e^2$ d) $-2\pi i e^{-1}$

16. If C is the circle $|z - 1| = 2$, then $\int \frac{\cot z}{z^3+z} dz$ is

- a) 0 b) $-4\pi i$ c) $2\pi i(1 - \cosh 1)$ d) $-2\pi i e^{-1}$

17. The singular point of $f(z) = \frac{z+1}{z^2(z^2+1)}$ are

- a) 0, i, -i b) 1, i, -i c) 0, i d) 1, -i

18. For $f(z) = \frac{e^{2z}}{(z-1)^3}$; $Z=1$ is a

- a) Removable singularity b) Pole of order 2
c) simple pole d) Essential singularity

19. For $f(z) = (z - 4)\sin \frac{1}{(z+3)}$; $Z = -3$ is a

- a) Removable singularity b) Pole of order 1
c) simple pole d) Essential singularity

20. For $f(z) = \frac{z - \sin z}{(z-3)}$; $Z = 0$ is a

- a) Removable singularity b) Pole of order 3
c) simple pole d) Essential singularity

21. The function $f(z) = \frac{e^z}{z}$, $z = 0$ is

- a) not a singular point b) a removable singularity c) simple pole d) essential singularity

22. The Taylor's expansion of $n \sin z$ about the origin is

- a) $1 + \frac{z^2}{2!} + \frac{z^4}{4!} + \dots$ b) $1 - \frac{z^2}{2!} + \frac{z^4}{4!} - \dots$ c) $1 + z + \frac{z^2}{2!} + \frac{z^3}{3!} + \dots$ d) $z - \frac{z^3}{3!} + \frac{z^5}{5!} - \dots$

23. Maclaurin's expansion $z - \frac{z^3}{3!} + \frac{z^5}{5!} - \dots$ is

- a) e^z b) $\cos z$ c) $\log(1+z)$ d) $\sin z$

24. In $|z| \leq 1$, the series $\sum \left(\frac{z^n}{n^2}\right)$ is

- a) convergent but not uniform b) divergent
c) uniformly convergent d) neither convergent nor divergent

25. in the open disc $|z| < 1$, then the series $1 + z + z^2 + z^3 + \dots$ is

- a) uniformly convergent b) convergent c) divergent d) absolutely convergent

26. The Taylor's expansion of $n \cos z$ about the origin is

- a) $1 + \frac{z^2}{2!} + \frac{z^4}{4!} + \dots$ b) $1 - \frac{z^2}{2!} + \frac{z^4}{4!} - \dots$ c) $1 + z + \frac{z^2}{2!} + \frac{z^3}{3!} + \dots$ d) $z - \frac{z^3}{3!} + \frac{z^5}{5!} - \dots$

27. The Taylor's expansion of $n \log(1+z)$ about the origin is

- a) $z + \frac{z^2}{2} + \frac{z^3}{3} + \dots$ b) $z - \frac{z^2}{2} + \frac{z^3}{3} - \dots$ c) $-z - \frac{z^2}{2} - \frac{z^3}{3} - \dots$ d) $z - \frac{z^3}{3!} + \frac{z^5}{5!} - \dots$

28. Taylor's series for $1/z$ about $z = 1$ is

- a) $1 + z + z^2 + z^3 + \dots$ b) $1 + (z-1) + (z-1)^2 + \dots$
c) $1 - (z-1) + (z-1)^2 - \dots$ d) $1 + \frac{z}{1!} + \frac{z^2}{2!} + \dots$

29. The expansion $\frac{1}{(z-1)(z-2)} = -\frac{1}{z} \left(1 + \frac{1}{z} + \frac{1}{z^2} + \dots\right) - \frac{1}{2} \left(1 + \frac{2}{z} + \left(\frac{2}{z}\right)^2 + \dots\right)$ is

- a) not a valued function b) valued in $1 < |z| < 2$ c) valued in $|z| < 1$ d) valued in $|z| > 2$

30. The only singularity of a single value function $f(z)$ are poles of order 1, 2 & at $z = -1$ & $z = 2$ with

residue at these poles 1 & 2 resp. $f(0) = \frac{7}{4}$ & $f(1) = \frac{5}{2}$ the function $f(z)$ is

- a) $1 + \frac{1}{1+z} + \frac{2}{z-2} + \frac{3}{(z-2)^2}$ b) $\frac{7}{4} + \frac{1}{1+z} + \frac{2}{z-2}$ c) $\frac{1}{1+z} + \frac{3}{(z-2)^2}$ d) $1 + \frac{2}{z-2} + \frac{3}{(z-2)^2}$

Answer key :

1	6	11	16	21	26
2	7	12	17	22	27
3	8	13	18	23	28
4	9	14	19	24	29
5	10	15	20	25	30

Test no : (Complex analysis – Previous Year Questions)

1. The amplitude of a complex number i is

- a) 0 b) $\pi/2$ c) π d) 2π

2. The amplitude of the quotient of two complex numbers is

- a) the sum of their amplitudes b) the difference of their amplitudes
c) the product of their amplitudes d) the quotient of their amplitudes

3. The $z = x + iy$ then $\bar{z} =$

- a) $x^2 + y^2$ b) $x^2 + iy^2$ c) $x^2 - y^2$ d) imaginary

4. If z_1 & z_2 are any two complex numbers, then $\frac{|z_1 + z_2|}{|z_1 z_2|}$ is

- a) $\frac{|z_1 + z_2|}{|z_2|}$ b) $\frac{|z_1 + z_2|}{|z_1|}$ c) $\frac{|z_1 + z_2|}{|z_2||z_1|}$ d) $\frac{|1|}{|z_1|} \frac{|1|}{|z_2|}$

5. The function $w = \frac{z^2+1}{z^2-1}$ is analytic

- a) for all z b) for all z except $1, -1$ c) for all z except $i, -i$ d) for all z except $1 + 2i$

6. Which of the following is not an entire function ?

- a) e^z b) e^{-z} c) $z^2 - 1$ d) $\log z$

7. If C is $|z| = R < 1$ then $\int \frac{z^2+1}{z^2-1} dz$ is

- a) 0 b) $2\pi i$ c) πi d) $2\pi i$

8. If $f(z)$ is an entire function & if $\int \frac{f(z)}{z-a} dz = 0$ then

- a) $z=a$ lies inside C b) $z=a$ lies on the boundary of C c) $z=a$ lies outside of C d) none

9. If $f(z)$ is analytic within & on C & $z=a$ lies inside C , then $\int \frac{f(z)}{z-a} dz$

- a) 0 b) $2\pi i f'(a)$ c) $\pi i f(a)$ d) $2\pi i$

10. Which of the following is called C-R equation for the function $f(z) = u + iv$ to be analytic at z ?

- a) $u_x = -v_y$ & $u_y = v_x$ b) $u_x = v_y$ & $u_y = -v_x$ c) $u_x = v_y$ & $u_y = v_x$ d) $u_x = -v_x$ & $u_y = v_y$

11. The Taylor's expansion of $n \cos z$ about the origin is

a) $1 + \frac{z^2}{2!} + \frac{z^4}{4!} + \dots$ b) $1 - \frac{z^2}{2!} + \frac{z^4}{4!} - \dots$ c) $1 + z + \frac{z^2}{2!} + \frac{z^3}{3!} + \dots$ d) $z - \frac{z^3}{3!} + \frac{z^5}{5!} - \dots$

12. The Taylor's expansion of $n \log(1+z)$ about the origin is

a) $z + \frac{z^2}{2} + \frac{z^3}{3} + \dots$ b) $z - \frac{z^2}{2} + \frac{z^3}{3} - \dots$ c) $-z - \frac{z^2}{2} - \frac{z^3}{3} - \dots$ d) $z - \frac{z^3}{3!} + \frac{z^5}{5!} - \dots$

13. If z_1 & z_2 are the image of the complex plane of two diametrically opposite point on the Riemann's sphere then $\overline{z_1 z_2}$ is

a) 0 b) 1 c) -1 d) i

14. in the open disc $|z| < 1$, then the series $1 + z + z^2 + z^3 + \dots$ is

a) uniformly convergent b) convergent c) divergent d) absolutely convergent

15. If C is the unit circle $|z| = 1$, then $\int \frac{z^2}{z-3} dz$ is

a) 0 b) $2\pi i$ c) π d) $3\pi i$

16. If C is the circle $|z| = 3$, then $\int \frac{e^z}{z-2} dz$ is

a) 0 b) $2\pi i$ c) $2\pi i e^2$ d) $2\pi i e^{-2}$

17. $\lim_{z \rightarrow 2i} (2x + iy^2)^2 =$

a) $8i$ b) $-8i$ c) 16 d) $4i$

18. The radius of convergence of $\sum \left(\frac{z^n}{n!}\right)$ is

a) ∞ b) e c) $1/e$ d) 1

19. The image of the strip $0 < x < 1$ under the transformation $w = iz$ is

a) $0 < u < 1$ b) $0 < v < 1$ c) $1 < u < 2$ d) $1 < v < 2$

20. If C is the circle $|z| = 1$, then $\int \frac{e^z}{z^5} dz$ is

a) $2\pi i/5$ b) $\pi i/12$ c) $2\pi i e^2$ d) $2\pi i e^{-2}$

21. If C is the circle $|z + 1| = 1$, then $\int \frac{3z^2 + 7z + 1}{z+1} dz$ is

a) 0 b) $2\pi i$ c) $3\pi i$ d) $-6\pi i$

22. The equation $|z + 4i| + |z - 4i| = 10$ represents

a) Circle b) ellipse c) parabola d) hyperbola

23. Maclaurin's expansion $z - \frac{z^3}{3!} + \frac{z^5}{5!} - \dots$ is

- a) e^z b) $\cos z$ c) $\log(1+z)$ d) $\sin z$

24. The invariant points of $w = \frac{z-15}{z-7}$ is

- a) 3,-5 b) 3,-3 c) 3,5 d) -3,-5

25. The bilinear transformation that map the points $z_1 = \infty, z_2 = i, z_3 = 0$ into $w_1 = 0, w_2 = i, w_3 = \infty$ is

- a) $w = 1/z$ b) $w = -1/z$ c) $w = z$ d) $w = iz$

26. If C is the circle $|z| = 1$, then $\int \frac{z^2+5z+6}{z-2} dz$ is

- a) 0 b) $20\pi i$ c) $2\pi i$ d) $40\pi i$

27. The Taylor's expansion of $\sin z$ about the origin is

- a) $1 + \frac{z^2}{2!} + \frac{z^4}{4!} + \dots$ b) $1 - \frac{z^2}{2!} + \frac{z^4}{4!} - \dots$ c) $1 + z + \frac{z^2}{2!} + \frac{z^3}{3!} + \dots$ d) $z - \frac{z^3}{3!} + \frac{z^5}{5!} - \dots$

28. In $|z| \leq 1$, the series $\sum \left(\frac{z^n}{n^2}\right)$ is

- a) convergent but not uniform b) divergent
c) uniformly convergent d) neither convergent nor divergent

29. Let $f(z) = \begin{cases} \frac{\operatorname{Re} z}{|z|}, & z \neq 0 \\ 0, & z = 0 \end{cases}$ and $g(z) = \begin{cases} \frac{x^3(1+i)-y^3(1-i)}{x^2+y^2}, & z \neq 0 \\ 0, & z = 0 \end{cases}$

- a) both $f(z)$ & $g(z)$ are continuous at the origin
b) both $f(z)$ & $g(z)$ are not continuous at the origin
c) $f(z)$ is continuous at the origin & $g(z)$ is not continuous at the origin
d) $f(z)$ is not continuous at the origin & $g(z)$ is continuous at the origin

30. If $f(z) = u + iv$ is analytic & $u = 2x(1-y)$ then $f(z) =$

- a) $2z + iz^2$ b) $2z - iz^2$ c) $z + iz^2/2$ d) $z - iz^2/2$

31. The multiplicative identity in C is

- a) (0,0) b) (1,1) c) (0,1) d) (1,0)

32. $\left(\frac{1+\cos\frac{\pi}{8}+isin\frac{\pi}{8}}{1+\cos\frac{\pi}{8}-isin\frac{\pi}{8}}\right)^8 =$

- a) $1+i$ b) $1-i$ c) 1 d) -1

33. If α is a complex number such that $\alpha^2 + \alpha + 1 = 0$ then $\alpha^{31} =$

- a) α b) α^2 c) 0 d) 1

34. If p is a complex number & $|q| = 1$ then $\left| \frac{p-q}{pq-1} \right|$ is

- a) i b) $-i$ c) 1 d) -1

35. If C is a closed curve enclosing the origin, then the value of $\int \frac{1}{z^2 e^z} dz$ is

- a) 0 b) $-2\pi i$ c) $2\pi i$ d) $4\pi i$

36. If $f(z) = u + iv$ is analytic & $v = 2xy$ then $f(z) =$

- a) $z^4 + c$ b) $z^2 + c$ c) $z + c$ d) $z - iz^2/2$

37. If $f(z) = (x^2 + ay^2 - 2xy) + i(bx^2 - y^2 + 2xy)$ is analytic, then the value of a, b are respectively

- a) $-1, 1$ b) $-1, 2$ c) $1, -1$ d) $2, -1$

38. The residue of $f(z) = \frac{4}{z^3(z-2)}$ at a simple pole is

- a) $1/5$ b) $1/4$ c) $1/3$ d) $1/2$

39. If C is the circle $|z| = 1$, then $\int \frac{z^2+5}{z-3} dz$ is

- a) 0 b) $2\pi i$ c) $2\pi i$ d) $-28\pi i$

40. The transformation $w = e^z$ transforms the imaginary axis in the Z axis into

- a) a unit circle with center at origin in the W plane
b) a circle of radius 2 with center at origin in the W plane
c) a straight line in the W plane d) an ellipse in the W plane

41. The bilinear transformation that map the points $z_1 = 0, z_2 = -1, z_3 = \infty$

into $w_1 = -1, w_2 = -2 - i, w_3 = i$ is

- a) $w = \frac{2+iz}{z+2}$ b) $w = \frac{2+iz}{z-2}$ c) $w = \frac{-2+iz}{z+2}$ d) $w = \frac{-2+iz}{z-2}$

42. The invariant points of $w = \frac{z+1}{2z+1}$ is

- a) $\pm \frac{1}{\sqrt{2}}$ b) $\pm \frac{1}{\sqrt{3}}$ c) $\pm \frac{1}{2}$ d) $\pm \frac{1}{3}$

43. The poles of $z \cot z$ are

- a) $2n\pi$ b) $n\pi$ c) $(2n+1)\pi$ d) $(2n-1)\pi$

44. If $u = 2x + x^2 - my^2$ is harmonic, then the value of m is

- a) 3 b) 2 c) 0 d) 1

45. The residue of $f(z) = \frac{z+1}{2(z-2)}$ at $z=2$ is

- a) 1/5 b) 1/4 c) 1/3 d) 3/2

46. $\lim_{z \rightarrow 2} \left(\frac{z^2 - 4}{z - 2} \right) =$

- a) 2 b) 4 c) 1 d) 8

46. If $f(z) = a(x^2 - y^2) + ibxy + c$ is differentiable at every point, the constants

- a) $2b=a$ b) $4b=a$ c) $2a=b$ d) $a=b$

47. The angle of rotation at $z = 1+i$ under the map $w = Z^2$ is

- a) 45 b) 0 c) 90 d) 180

48. Taylor's series for $1/z$ about $z = 1$ is

- a) $1 + z + z^2 + z^3 + \dots$ b) $1 + (z - 1) + (z - 1)^2 + \dots$
 c) $1 - (z - 1) + (z - 1)^2 - \dots$ d) $1 + \frac{z}{1!} + \frac{z^2}{2!} + \dots$

49. The function $f(z) = |z|^2$ is

- a) differentiable at $z = 0$ b) no where differentiable c) not harmonic d) differentiable at $z \neq 0$

50. If the function $f(z) = \frac{z}{1+z}$, $u + iv =$

- a) $\frac{x}{x^2+y^2} - \frac{iy}{x^2+y^2}$ b) $\frac{x^2+y^2+x}{(x+1)^2+y^2} + \frac{iy}{(x+1)^2+y^2}$ c) $\frac{x^2+y^2}{(x+1)^2+y^2} + \frac{iy}{(x+1)^2+y^2}$ d) $(x^2 + 5) + 7yi$

51. The image of the circle $|z - 3i| = 3$ under the map $w = 1/z$ is

- a) circle b) st.line c) square d) rectangle

52. If C is $|z| = 1$ then $\int_C \frac{e^z}{z^n} dz$ is

- a) 0 b) $2\pi i$ c) πi d) $\frac{2\pi i}{(n-1)!}$

53. The singular point of $f(z) = \frac{z+1}{z^2(z^2+1)}$ are

- a) 0, i, -i b) 1, i, -i c) 0, i d) 1, -i

54. Any two harmonic conjugates of a given harmonic function $u(x,y)$ differ by

- a) x b) y c) xy d) constant

55. The function $f(z) = \frac{e^z}{z}$, $z = 0$ is

- a) not a singular point b) a removable singularity c) simple pole d) essential singularity

53. The residue of $f(z) = \frac{z+1}{z^2-2z}$ are

- a) 1/2 b) -1/2 c) 3/2 d) -3/2

54. The radius of convergence of $\sum \left(1 + \frac{1}{n}\right)^{n^2}$ is

- a) ∞ b) e c) $1/e$ d) 0

55. The bilinear transformation that map the points $z_1 = 0, z_2 = -i, z_3 = -1$ into $w_1 = i, w_2 = 1, w_3 = 0$ is

- a) $w = (i) \frac{z+1}{z-1}$ b) $w = (-i) \frac{z+1}{z-1}$ c) $w = (i) \frac{z-1}{z+1}$ d) $w = (-i) \frac{z-1}{z+1}$

56. If C is the positively oriented circle $|z - i| = 2$, then $\int \frac{e^z}{z^2+4} dz$ is

- a) πe^{2i} b) $\frac{\pi}{2}(e^{2i} - e^{-2i})$ c) $\frac{\pi}{2}(e^{2i})$ d) $2\pi e^{2i}$

57. The transformation $w = 1/z$ transforms

- a) circle not through the origin in the z- plane into the circle through the origin in the w-plane
- b) circle through the origin in the z- plane into the st.line passing through the origin in the w-plane
- c) st.line not through the origin in the z- plane into the circle through the origin in the w-plane
- d) st.line through the origin in the z- plane into the st.line passing through the origin in the w-plane

58. The only singularity of a single value function $f(z)$ are poles of order 1,2 & at $z = -1$ & $z = 2$ with residue at these poles 1 & 2 resp. $f(0) = \frac{7}{4}$ & $f(1) = \frac{5}{2}$ the function $f(z)$ is

- a) $1 + \frac{1}{1+z} + \frac{2}{z-2} + \frac{3}{(z-2)^2}$ b) $\frac{7}{4} + \frac{1}{1+z} + \frac{2}{z-2}$ c) $\frac{1}{1+z} + \frac{3}{(z-2)^2}$ d) $1 + \frac{2}{z-2} + \frac{3}{(z-2)^2}$

59. The expansion $\frac{1}{(z-1)(z-2)} = -\frac{1}{z} \left(1 + \frac{1}{z} + \frac{1}{z^2} + \dots\right) - \frac{1}{2} \left(1 + \frac{2}{z} + \left(\frac{2}{z}\right)^2 + \dots\right)$ is

- a) not a valued function b) valued in $1 < |z| < 2$ c) valued in $|z| < 1$ d) valued in $|z| > 2$

Answer key :

1	11	21	31	41	51
2	12	22	32	42	52
3	13	23	33	43	53
4	14	24	34	44	54
5	15	25	35	45	55
6	16	26	36	46	56
7	17	27	37	47	57
8	18	28	38	48	58

9	19	29	39	49	59
10	20	30	40	50	60