

ALGEBRA

TEST :

1) Let C be the set of ordered pairs (a, b) of real numbers and multiplication in C are defined by the equations

$(a, b) + (c, d) = (a + c, b + d)$ $(a, b) \cdot (c, d) = (ac - bd, bc + ad)$ Then the multiplicative identity is

- A) $(0, 0)$ B) $(1, 1)$ C) $(0, 1)$ D) $(1, 0)$

2) Which one of the following is a Ring without unity?

- A) The set of all integers with respect to addition and multiplication.
B) The set of all even integers with respect to addition and multiplication.
C) The set of all rational with respect to addition and multiplication.
D) The set of all real with respect to addition and multiplication.

3) Which one of the following is not an integral domain?

- A) The set of all integers with respect to addition and multiplication
B) The set of all rational with respect to addition and multiplication.
C) The set of numbers of the form $a + b\sqrt{2}$ with a and b as integers with respect to addition and multiplication.
D) The set of all ordered pairs (a, b) of real numbers with respect to addition and multiplication defined by the equations.

$$(a, b) + (c, d) = (a + c, b + d)$$

$$(a, b) \cdot (c, d) = (ac, bd)$$

4) Which one of the following set of vectors in $F^{(3)}$ is linearly dependent?

- A) $\{(1,0,0), (0,1,0), (0,0,1)\}$ B) $\{(1,1,0), (3,1,3), (5,3,3)\}$
C) $\{(-1,2,1), (3,1, -2)\}$ D) $\{(2,3,5), (4,9,25)\}$.

5) Let F be any field and K be a subfield of F . R be the field of real numbers and C the field of complex numbers. Then which one of the following statements is not correct?

- A) F is a vector space over F B) F is a vector space over K
C) R is a vector space over C D) C is a vector space over R

6) The value of a for which the set of vectors

$\{(1 + a, 1,), (2 + a, 2 + a, 2), (3 + a, 3 + a, 3 + a)\}$ is not a basis for R^3 is

- A) 0 B) 1 C) 2 D) 3.

7) Which one of the following statements is wrong?

- A) Every group G with identity e such that $x^2 = e$ for all $x \in G$ is Abelian
- B) If G is an Abelian group then for all $a, b \in G$ $(ab)^n = a^n b^n$
- C) If every element of a group G is its own inverse then G is Abelian.
- D) If G is an Abelian group then every element of G is its own inverse.

8) Which one of the following is not a group?

- A) The set of all $n \times n$ matrices with their elements as integers with respect addition of matrices.
- B) The set of all $n \times n$ non-singular matrices with their elements as integers with respect to multiplication of matrices.
- C) The set of all $n \times n$ matrices with their elements as rational with respect to matrix addition.
- D) The set of all $n \times n$ non-singular matrices with their elements as rational with respect to matrix multiplication.

9) $\{1, -1, i, -i\}$ is a group with respect to multiplication. The respective orders of the elements $-1, -i$ are.

- A) 2, 2 B) 4, 2 C) 2, 4 D) 4, 4.

10) Which one of the following statements is wrong?

- A) A non-empty subset H of the group G is a subgroup of G if and only if $a \in H, b \in H \Rightarrow ab \in H$ and $a \in H \Rightarrow a^{-1} \in H$.
- B) A necessary and sufficient condition for a non-empty subset H of a group G to be a subgroup is that $a \in H, b \in H \Rightarrow a^{-1} \in H$.
- C) The intersection of two subgroups of a group is also a subgroup.
- D) union of two subgroups of a group is also a subgroup.

11) If G is group of order 8, then G can have a subgroup of order.

- A) 2 B) 3 C) 5 D) 7.

12) A ring is called a Boolean ring, if

- A) $a^2 = e$ for all $a \in R$ (Where e is the multiplicative identity)
- B) $a^2 = 0$ for all $a \in R$ C) $a^2 = a$ for all $a \in R$ D) $a^n = 0$ for some $n \in \mathbb{N}$

13) An infinite commutative ring without identity element is

- A) $(\mathbb{Z}, +, \cdot)$ B) $(\mathbb{Z}_n, +_n, \cdot_n)$ C) $(2\mathbb{Z}, +, \cdot)$ D) $M_2(\mathbb{R})$

14) The product of $2x + 4$ and $4x^2 + 3x + 1$ in $\mathbb{Z}_5[x]$ is

- A) $8x^3 + 22x^2 + 14x + 4$ B) $8x^3 + 2x^2 + 4x + 4$

C) $3x^3 + 2x^2 + 3x + 4$

D) $3x^3 + 2x^2 + 4x + 4$

15) The basis for the vector space $V = \left\{ \begin{pmatrix} a & b \\ 0 & d \end{pmatrix} / a, b, d \in R \right\}$ is

- A) $\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$ B) $\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \right\}$
 C) $\left\{ \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \right\}$ D) $\left\{ \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$

16) Any ordered integral domain is characteristic of

- A) 1 B) 0 C) ∞ D) Prime.

17) Let $G = \left\{ \begin{bmatrix} x & x \\ x & x \end{bmatrix}, x \in R^* \right\}$. Now $(G, +)$ is

- A) A group with identity element $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ B) Not a group
 C) An Abelian group D) A non-Abelian group

18) The order of [7] in $(Z_9, +_9)$ is

- A) 1 B) 3 C) 6 D) 9

19) "If G is a finite group and H is any sub-group of G , then the order of H divides order of G ". This is known as

- A) Cayley's theorem B) Lagrange's theorem
 C) Euler's theorem D) Fermat's theorem

20) Let G be a group $a, b \in G$, then $(b^{-1} * a * b)^3$ equals

- A) $b^{-1} * a^3 * b$ B) $(b^{-1})^3 * a^3 * b^3$ C) $b^{-1} * a * b^3$ D) $(b^{-1})^3 * a * b^3$

21) If A and B be sub-groups of a finite group G such that A is a sub-group of B , then

- A) $[G:A][A:B] = [G:B]$ B) $[G:B][B:A] = [G:A]$
 C) $[G:B][A:B] = [G:A]$ D) non of these.

22. If A and B are two finite subgroup of group G , then

- A) $[G:A] = [G:B][B:A]$ B) $[G:B] = [G:A][A:B]$
 C) $|AB| = \frac{|A||B|}{|A \cap B|}$ D) $|AB| = \frac{|A||B|}{|A \cup B|}$

23. Which of the following tables can represent a group?

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24) The Set $R = \left\{ \begin{bmatrix} a & b \\ -b & a \end{bmatrix} \mid a, b \in R \right\}$ is a ring under matrix addition and multiplication.

The inverse of $\begin{bmatrix} a & b \\ -b & a \end{bmatrix}$ is

- A) $\begin{bmatrix} -a & b \\ -b & a \end{bmatrix}$ B) $\begin{bmatrix} a & -b \\ -b & a \end{bmatrix}$ C) $\begin{bmatrix} a & +b \\ -b & -a \end{bmatrix}$ D) $\begin{bmatrix} -a & -b \\ b & -a \end{bmatrix}$

25) If a non-empty subset W of a vector space V over a field F is a subspace of V , then $\alpha, \beta \in F$ and $u, v \in W \Rightarrow$

- A) $\alpha u + \beta v \in W$ B) $\alpha u + \beta v \in W$ C) $\alpha u + \beta v \in V$ D) $\alpha u \in W$

26) Which one of the following sets of vectors is linearly dependent?

- A) $\{(1,4, -2), (-2,1,3), (-4,11,5)\}$ B) $\{(1,2,1), (2,1,0), (1, -1,2)\}$
 C) $\{(1,0,0), (0,1,0), (1,1,0)\}$ D) $\{(0,0,0), (2,5,3), (-1,0,6)\}$

27) Which one of the following sets of vectors is not a basis for $V_3(R)$?

- A) $\{(1,0,0), (1,1,0)\}$ B) $\{(1,0,0), (0,1,0), (0,0,1)\}$
 C) $\{(1,0,0), (0,1,0), (1,1,1)\}$ D) $\{(1,1,0), (0,1,1), (1,0,1)\}$

28) $G = \left\{ \begin{bmatrix} x & x \\ x & x \end{bmatrix} \mid x \in R^* \right\}$ is a group under matrix multiplication. The identity element of G is

- A) $\begin{bmatrix} \frac{x}{4} & \frac{x}{4} \\ \frac{x}{4} & \frac{x}{4} \end{bmatrix}$ B) $\begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$ C) $\begin{bmatrix} \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} \end{bmatrix}$ D) $\begin{bmatrix} \frac{x}{2} & \frac{x}{2} \\ \frac{x}{2} & \frac{x}{2} \end{bmatrix}$

29) If A and B are subgroups of an Abelian group G , then

- A) $A \cup B$ is a subgroup of G B) $A \times B$ is a subgroup of G
 C) AB is a subgroup of G D) $A * B$ is a subgroup of G

30) If H is a subgroup of G and N is a normal subgroup of G , then

- A) HN is a subgroup of G B) H is a normal subgroup of G
 C) $A \cap B$ is a normal subgroup of G D) $H \cup N$ is a normal subgroup of G

31) Which one of the following is a vector space? $R \times R$ with usual addition and scalar multiplication defined by

- A) $\alpha(a, b) = (0, \alpha b)$ B) $\alpha(a, b) = (\alpha a, \alpha^2 b)$
 C) $\alpha(a, b) = (\alpha a, \alpha b)$ D) $\alpha(a, b) = (|\alpha|a, |\alpha|b)$

32) If A and B are two subspaces of a vector space V over a field F , then

- A) $A \cup B$ is a subspace of V B) $A \times B$ is a subspace of V
 C) $A \cap B$ is a subspace of V D) AB is a subspace of V